On Target Detection by Quantum Radar – Supplementary Information

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Abstract. *In a large part of the pertaining literature one founds an alleged "quantum advantage" of a "Quantum Radar" (QR) over the conventional (or "classical") radar (CR). In reality, the maximum Range of a microwave quantum radar for typical aircraft targets is limited by the laws of physics to meters or tens of meters. The detection performance of all the proposed QR types are orders of magnitude below the ones of any much simpler and cheaper equivalent "classical" radar set, in particular of the Noise Radar (NR) type. The pertaining result, based on the laws of physics, shows notable sociological facets, particularly in the field of the behaviour of researchers and their groups.*

List of Acronyms

CMB	Cosmic Microwave Background
CNIT	National Inter-University Consortium for Telecommunications
CR	Classical Radar – Conventional Radar
DRFM	Digital Radio Frequency Memory
FMCW	Frequency Modulated Continuous Wave
IEEE	Institute of Electrical and Electronics Engineers
INRIM	Istituto Nazionale di Ricerca Metrologica (National Metrology Institute-Italy)
ITU	International Telecommunication Unit (an United Nations Agency)
JPA	Josephson Parametric Amplifier
JTWPA	Josephson Traveling Wave Parametric Amplifier
MQI	Microwave Quantum Illumination
NR	Noise Radar
PAPR	Peak-to-Average Power Ratio
PRN	Pseudo-Random Number
PSL	Peak Sidelobe Level
QI	Quantum Illumination
QKD	Quantum Key Distribution
QR	Quantum Radar
QTMS	Quantum Two-Mode squeezed
QTR	Quantum Target Ranging
RaSS	Radar and Surveillance Systems
RCS	Radar Cross Section
R&D	Research and Development
ROC	Receiver Operating Characteristic
RTG	Research Task Group
SET	Sensors and Electronic Systems
SI	*Système International d'Unités*, International System of Units
SNR	Signal-to-Noise Ratio
SPDC	Spontaneous Parametric Down Conversion
SQUID	Superconducting QUantum Interference Device
SWaP	Size, Weight and Power
TMN	Two-Mode Noise
TMSV	Two-Mode Squeezed Vacuum
TRL	Technology Readiness Level

1. Introduction

1.1. The International Research Context

The last decades have seen theoretical and experimental research activities aimed to apply *quantum technologies* in fields showing potential interest, such as – listed from the oldest to the newest ones – *cryptography*, *computers*, *communication networks* and *sensing*.

The birth of *Quantum Cryptography* can be dated back to 1984 with the publication of the celebrated Bennet & Brassard (BB84) method for cryptographic key distribution [1]. However, in spite of the elapsed forty years (a time frame which has seen enormous progresses in processing, sensing and telecommunications), practical and industrial applications of Quantum Key Distribution (QKD) and, more generally, of Quantum Cryptography have been much more limited than one could expect two decades ago. In fact, the infrastructure required for quantum communication (including quantum networks and quantum devices) is not widely available and Quantum Cryptography is not yet widely available to end users, being primarily pursued and tested by academic and governmental organizations, and by some industry players, with limited deployments in certain niche applications and for a small number of users. Probably the major effect of research on Quantum Cryptography were some important investments on the related fields of Quantum Computation and Quantum Networks, see for example *"Quantum Internet Alliance-Phase 1"* (1 October 2022 – 31 March 2026, Total 24 M€), [2]. According to Boston Consulting Group, the investments on Quantum Computing area in 2022 have reached 1.6 billion dollars.

Nowadays, the most effective and practical application of Quantum Mechanics is found in Metrology [3], with the redefinition of the international system (SI) approved by the General Conference of Weights and Measures on 16 November 2018 and entered into force on 20 May 2019. All seven base units (defined in terms of a fundamental constant of nature) and *volt*, *ohm* and *ampere* are realized by quantum experiments in solid-state devices. This Conference fixed the *exact* values for the electron charge e and the Planck's constant h as:

$$e = 1.602176634 \cdot 10^{-19}\, A \cdot s \qquad h = 6.62607015 \cdot 10^{-34}\, J \cdot s.$$

The huge *private* investments related to quantum technologies are synthesized in Fig. 1 and in Table 1 (from [4]). Around the 2022 peak, the order of magnitude of public investments – including research funding – is twice that of venture capital, i.e. reaches the remarkable amount of $ 4-5 billion yearly.

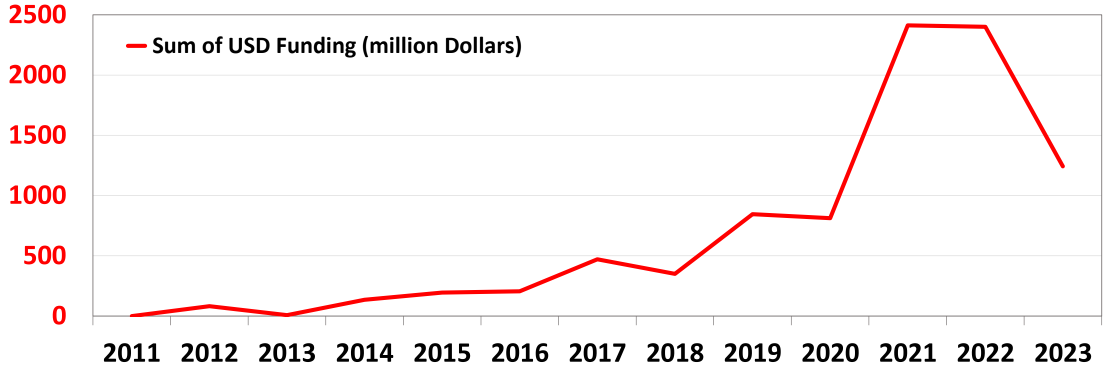


Figure 1. Sum of private US $ funding in $ million (source: The Quantum Insider, updated end of December 2023, [4].

Table 1. Total private investment by region ($ million). EMEA: Europe, Middle East, and Africa; APAC: Asia-Pacific (source: The Quantum Insider, updated end of December 2023, [4]).

	AMERICAS	EMEA	APAC	TOTAL
2022	1369	762	260217	2391
2023	240	781		1238

It may be interesting to consider the quantum-technology researches in the frame of the story of technical and industrial developments since the (first and second) Industrial Revolution, when most of the related technical and scientific developments have been conceived and implemented in order to *solve real world problems* according to the *"problem solver"* paradigm. Conversely, some recent research and development (R&D) efforts appear to belong to the opposed paradigm of *"a solution calling for a problem"*. Quantum Radar (and microwave Quantum Illumination – QI) are an example, as discussed later on.

Nevertheless, Quantum Computer (QC) is a best-known example of *a solution calling for a problem*, as discussed in [5]. This is clearly confirmed by one of the main private founders of QC, that is, Google. The Google-GESDA-XPRIZE (https://www.xprize.org/prizes/qc-apps) is a 36 months, five million dollars global competition for defining quantum computer applications to solve real world problems: difficult not to see in it the paradigm of *a solution calling for a problem*.

From another point of view, Quantum Computer is allegedly forty years old [6] and, still, no a single effective general-purpose set is on sale for the public. In fact, after the fanfare about the quantum algorithm for Shor's factorization in 1994, no more algorithms with similar potential were found. In forty years another technology, the one of mobile/personal communications, has progressed in five generations i.e. from the analogue 1G (e.g. the forgotten TACS of 1979) to the widely known 5G (from 2018 till 2024, waiting for 6G).

More generally, the recent years have seen the birth of a sort of a worldwide *"quantum fever"* affecting many domains, arriving to surprising proposals such as heat engines operating at $T \cong 120\, nK$ [7], and some endoreversible quantum Otto cycle engines [8].

In this *"fever"*, as one could expect, some researchers in the quantum mechanics area could not *"sociologically"* remain at the window and tried to propose new developments in the (brand new to them) radar area, avoiding radar engineers in their research teams. Then, since about fifteen years it was, and it is sometimes claimed, that a Quantum Radar (QR) has the potential to outperform classical radar thanks to the properties of quantum mechanics, including in some cases, the detection of *stealth targets* [9], [10], [11], [12] at a long-distance [13].

Moreover, in [14] we read (comments are left to the reader): *"Billions of dollars have already been spent on quantum computing; compared to this, a few million dollars to develop a field-testable QTMS radar does not seem extravagant. With the evidence before us, it seems worthwhile to make a modest effort to understand the possibilities of quantum radars more thoroughly"*.

Hence, we quote from [10]: *"Canada has also invested C$ 2.7 m (£ 1.93 m) into developing quantum radar via an ongoing research project at the University of Waterloo"*.

As a *"national"* example, the Italian Ministry of Defense – thru his organization named *Teledife* (Direzione Informatica Telematica e Tecnologie Avanzate – Amministrazione Difesa: https://www.difesa.it/sgd-dna/staff/dt/teledife/index/30197.html) is financing the research project *"Quantum Radar"* (30/04/2022 – 01/05/2025), [15].

1.2. Outline and scope of this work

This work, written for engineers or managers with no special knowledge of quantum mechanics, aims to help the scientific, industrial and institutional community to understand better the engineering aspects and the basic limitations of the proposed QR technologies and demonstrators.

For reasons the reader will understand soon, we start with two sentences from the *"Manual of Political Economy"* by Vilfredo Pareto (1848 – 1923):

"When it is useful to them, men can believe a theory of which they know nothing more than its name." and: *"Men follow their sentiments and their self-interest, but it pleases them to imagine*

that they follow reason. And so, they look for, and always find, some theory, which, a posteriori, makes their actions appear to be logical. If that theory could be demolished scientifically, the only result would be that another theory would be substituted for the first one, and for the same purpose".

Detection and ranging capabilities for QR are critically discussed and a comparison with its *closest* Classical Radar is presented. Finally, conclusions are drawn on the potential advantages of a QR versus its Noise Radar counterpart and, more generally, on the QR research.

In particular, it is investigated whether a future fielded and operating QR system might really outperform an *"equivalent" classical* radar, or not (here, the term *"classical"* is opposed to *"quantum"* and does include advanced radar architectures such as the Noise Radar one).

Moreover, it is investigated whether (or not) this QR could reasonably detect a target outside a laboratory, i.e. at least at hectometer or kilometer ranges, such as the ones of a cheap (order of thousand €) and simple marine radar similar to the one whose characteristics and performance are shown in [16].

The numerical evaluations of this work only consider the power budget for the Radar Range computations. For the other aspects, although equally relevant (such as accuracy, resolution, low probability of intercept, resilience to the jamming, etc.) the interested reader is addressed to [17], [18] and [19], rich in explanations and in References. The characteristics of certain proposed types (see for instance [14] and [20]) of Quantum Radar (sometimes the term *"quantum protocol"* is used in place of *"type"*) make them allegedly similar to a Noise Radar (NR), and a clarification is done in this frame, too.

For the interested reader, an informal overview on the basic concepts in Quantum Mechanics is presented in Chapter 2 of [21], a book fully of nice, useful criticism, while a complete treatment of QM with many examples is available in [22].

An updated analysis of QR operation and performance with an attempt to cope with quantum, classical and engineering points of view is presented in [23].

The comprehensive paper [24] shows a basic overview of quantum technology for military applications, also estimating the expected time scale of delivery or the utilization impact, i.e. the Technology Readiness Level (TRL). Table 2 – from [24] – shows the TRL and the time horizon expectations.

Table 2. TRL and Time Horizon expectations, from [24].

Technology	TRL	Time Horizon
Quantum computer (annealer)	4-5 (5-6)	2030
QKD (satellite)	7-8 (6-7)	2025 (2030)
Post-quantum cryptography	7-8	2025
Quantum communication network	1-3	2030-2035
Quantum inertial navigation	4-5	2025-2030
Quantum clocks	4-6	2030
Quantum radar	**1-2**	**None**
Quantum RF antenna	4	2025-2030
Quantum magnetic and gravity sensing	5-6	2025
Quantum imaging	5	2025-2030

TRL levels are defined as: (1) basic principles observed; (2) technology concept formulated; (3) experimental proof of concept; (4) technology validated in lab; (5) technology validated in relevant environment (industrially relevant environment in the case of key enabling technologies); (6) technology demonstrated in relevant environment (industrially relevant environment in the case of key enabling technologies); (7) system prototype demonstration in operational environment; (8) system complete and qualified; (9) actual system proven in operational environment (competitive manufacturing in the case of key enabling technologies or in space).

1.3. Literature and Technical Evaluations on Quantum Radar

A search for *"quantum radar"* on the well-known IEEE database IEEEXplore (https://ieeexplore.ieee.org/Xplore/home.jsp), which includes more than 6.1 million items, yielded the results shown in Fig. 2. They amount to 132 papers (starting from 2013) distributed on Journals (28), Magazines (20) and Conference Proceedings (84). Publications grew rapidly from 2017 to 2020, when they peaked and then quickly declined. Including the papers not referenced in IEEEXplore, the total largely exceeds a few hundred publications, including one book, [25]. An interesting result of the literature analysis, see also [26] and [27], is that this book is *antecedent* to all the IEEE found papers and to most of the overall papers, while according to the common behaviour of scientific publications, a book is expected to be *subsequent* to the most significant papers on the same topic.

Analyzing the most important international Radar Conference of the last years, the topic "Quantum Radar" appears in the conference topics of the European Radar Conferences (EuRad 2022, 2023 and 2024); however, only one (invited) paper [28] is found in the Proceedings of the Conferences. In the incoming EuRad 2025, *Quantum Radar* has been included into the larger topic of Ultra-Wideband, Noise and Polarimetric Radar.

A session on Quantum Radar (titled 'Quantum radar: real world experiments and new theory') appeared for the first time in 2020 at the IEEE Radar Conference (21-25 September 2020, Florence, Italy), including five papers. In addition, another paper on QR was presented in a different Session (Radar Technology) [29]. In the IEEE Radar Conference (7-14 May 2021, Atlanta, GA, USA) a Special Session titled 'Quantum Radar Theory and Practice (Part I and Part II) has eight papers.

The years 2020 and 2021 show the maximum interest concerning Quantum Radar, as pointed out in Fig. 2. In 2023 and 2024 the interest on QR is rapidly decreasing: in 2023, a single paper [13] was presented at the IEEE Radar Conference in San Antonio, TX (1-5 May 2023). In 2024 only one paper [30] has been presented at the International Radar Symposium (IRS 2024) 2-4 July 2024, Wroclaw, Poland.

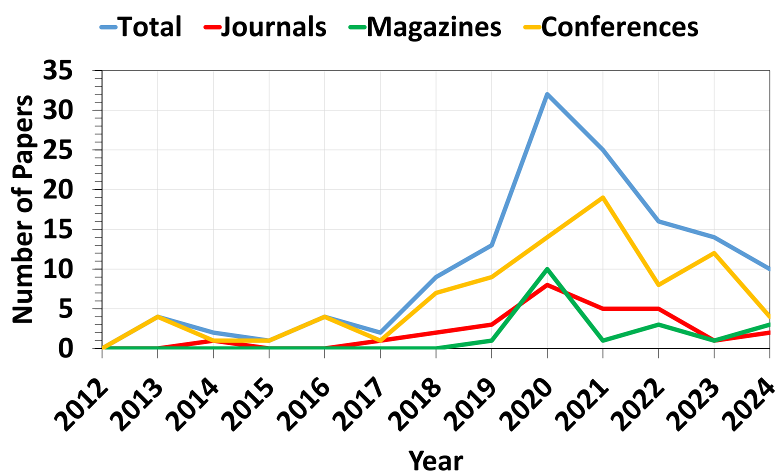


Figure 2. Papers on Quantum Radar year by year referenced in IEEEXplore (From: https://ieeexplore.ieee.org/Xplore/home.jsp).

Most QR papers are oriented to quantum physics and to technological aspects, with a very few contributions (order of a few %) considering system (and of course, operational) concepts. Operating and outdoor demonstrators are practically absent: sometimes, in spite of the acronym RADAR (Radio Detection and Ranging), some described laboratory tests refer to optical wavelengths (i.e. to a Lidar, not to a Radar); anyway, the tests are in the lab, not outdoor.
In one case the *"quantum radar demonstrator"* – as shown in Fig. 3 of [20] – is made up by a transmitter connected to a horn antenna facing another (receiving) horn antenna at a distance of less than one meter (both horns being fixed with adhesive tape on a desk, neglecting multipath and reflection effects), i.e. with no radar target at all. Hence, the experiment shown in [20] was carried out with a two-way propagation attenuation a_R (see section 5.4) not measured but probably close to the unity. Conversely, in the aforementioned literature one founds an even lower attenuation: see for instance, among the recent papers, [31], where the results, shown in Figs. 2 and 3, are obtained, according to their captions, with a round-trip attenuation of mere two orders of magnitude ($-20\ dB$), an unrealistic value found in most publications on QR. This attenuation corresponds to a radar Range of the order of *one meter* (in the X-band, using horn antennas and for a target of $1\ m^2$ radar cross section). Note that in a real radar operation, the two-way attenuation at X-band is of the typical order of 10^{13} as a ten watts transmitted and a one square meter target RCS generate echoes of the typical order of a Pico Watt (or less) at normal (kilometric) target distances (monostatic radar, antenna gain between 20 and 30 dB), see for instance Fig. 9.
Some widely known papers on QR lead us to emphasize the importance of the Order of Magnitude and of the selection of the Most Suited Model and finally, the Common Sense. In fact, from the aforementioned literature, these canonical concepts seem not to be always obvious. The related considerations are deemed useful and briefly resumed in section 7.

2. Classical Radar, Noise Radar and Quantum Radar

It is well known that radar detection [32], [33], [66] depends on the *energy* received from the target's backscattering rather than on its power, as the output of the optimum (or *"matched"*) filter [34], is proportional to the *energy* of the received waveform divided by the spectral density of the noise E/N_0. Hence, most *classical* radar waveforms have a constant amplitude (i.e. are phase-coded) in order to exploit at best the power amplifier, granting a Peak-to-Average Power Ratio (PAPR), [35], [36], [37] equal to the unit to maximize, with the bound of the maximum transmittable power, the received total energy in the dwell time.

A Classical Radar (CR) operation is shown in Fig. 3a. In the case of a pulse compression radar, the received signal is correlated (nowadays, in the Fourier domain) with a template of the transmitted one.
In Noise Radar (NR), see Fig. 3b, the received signal is correlated with a template (*reference*) of the transmitted one, which is created by a (possibly, *"tailored"*) realization of a random process, in turn, obtained by a noise source or, more frequently, by a pseudorandom numbers generator (see section 3.2).
In QR, see Fig. 3c, the received signal is correlated with the *idle signal* entangled with of the transmitted one, which is a realization of a random process. The fact that the signal realizations in QR cannot be controlled (hence, cannot be tailored) is a key difference between QR and NR, whose analysis is presented in an ensuing section.

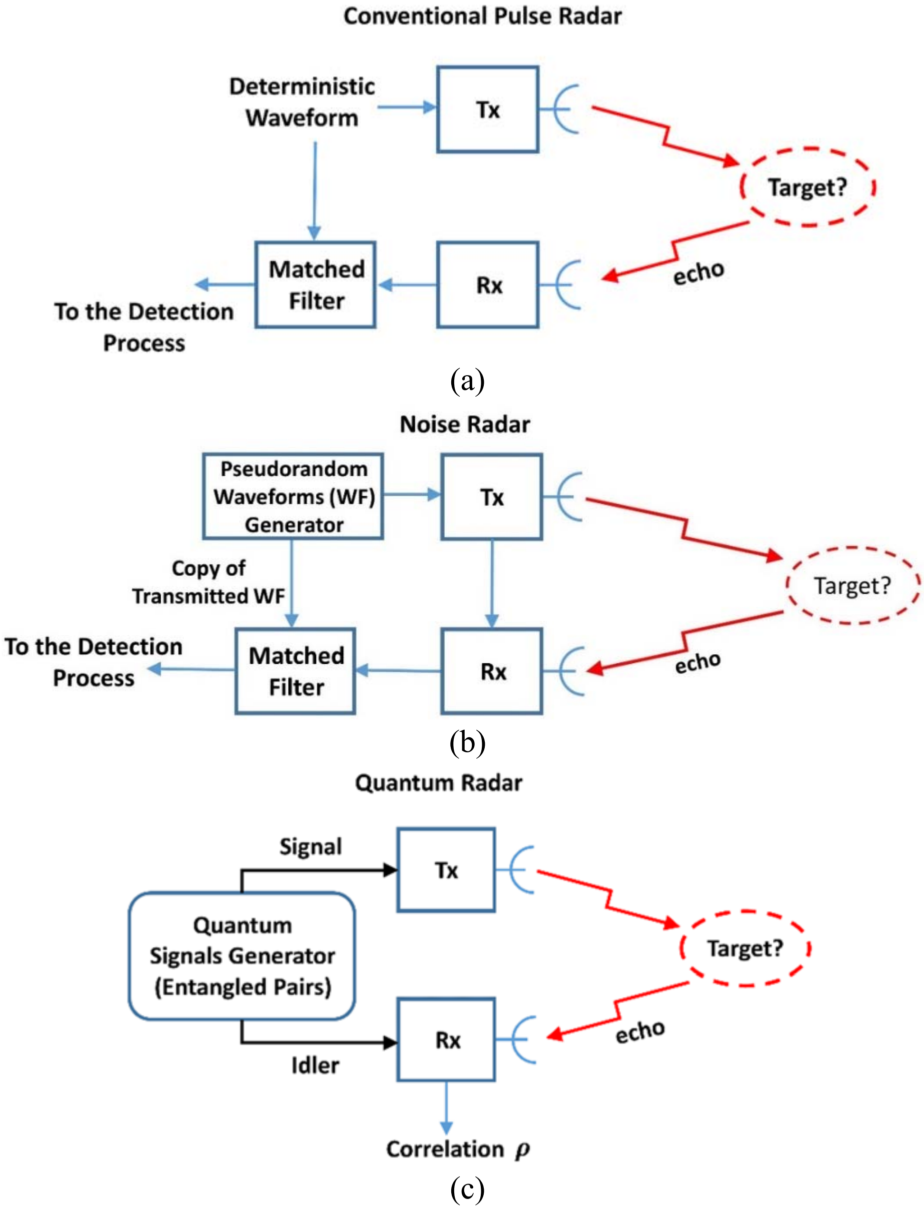


Figure 3. Block-diagram comparison of: (a) Conventional Radar (CR), (b) Noise Radar (NR) and (c) Quantum Radar (QR).

3. Brief Introduction to Noise Radar

3.1. A Short History of Noise Radar

The history of Noise Radar (NR) is quite old; Horton introduced the NR in 1959 for a high-resolution distance measurement system. The generation of noise signals was first implemented using *analog sources*. Research was performed in China, in USA, and in Europe. Since the 2000s, experimental research activity with field trials took place in Ukraine, where in 2002, the First International Workshop on Noise Radar Technology (NRTW 2002) was held.

In Poland, at the Warsaw University of Technology starting from 2010's, a NR demonstrator was implemented using commercial hardware for the detection of moving targets at short (hundred meters) range. From 2005, a noticeable pre-competitive and unclassified research effort on NR is being developed in the frame of the NATO Sensors and Electronic Systems (SET) Research Task Groups (RTG's). The related results are shown in the official reports of RTG SET-101 "Noise Radar Technology" (2005-2008), RTG SET-184 "Capabilities of Noise Radar" (2012-2014) and RTG SET-225 "Spatial and Waveform Diverse Noise Radar" (2015-2018). From December 2020 the activities continue within the RTG SET-287 "Characterization of Noise Radar".

3.2. Noise Radar Architecture and Waveforms Generation

In modern NR the preferred solution to generate the transmitted waveform is fully digital and uses pseudo-random number (PRN) generators, see for instance [38], [39], [40], [41] and [42].

The high-level block diagrams of a NR is shown in Fig. 4. The *reference* signal is the record of the transmitted signal at the antenna port.

The randomness of the transmitted waveform normally generates $PAPR$ greater than one, statistically above three (i.e. 10 – 12 dB), with a significantly reduction of the signal energy, hence affecting the detection performance [35].

The resulting loss L in $[dB]$ versus $PAPR$, i.e. $L = -10 \cdot \log_{10}(PAPR)$, is shown in Fig. 5.

Moreover, radar operation with time-bandwidth product $BT > 1$ poses the problem of Range-sidelobes at the output of the coherent integration (i.e. of the compression filter). In NR, the problem may be faced by a suited *"tailoring" of the transmitted signals* (as shown in Fig. 6, [42]), allowing a Peak Sidelobe Level (PSL) as low as (typically) -60 dB (or better) below the main lobe in the normalized autocorrelation function.

An example of the normalized autocorrelation of a noise waveform, obtained by the waveform generator of Fig. 6, is shown in Figs. 7 (blue line), where the PAPR is equal to 1.5 corresponding to a 1.76 dB loss (pink region in Fig. 5). In QR the signal is *"naturally Gaussian"* random and not modifiable (as in Figs. 7, red line), hence the sidelobes are not controlled and the resulting $PAPR$ (around 10 or greate: green area in Fig. 5), gives a loss close to -10 dB. Such an intrinsical loss of every QR definitely overcomes any "quantum advantage".

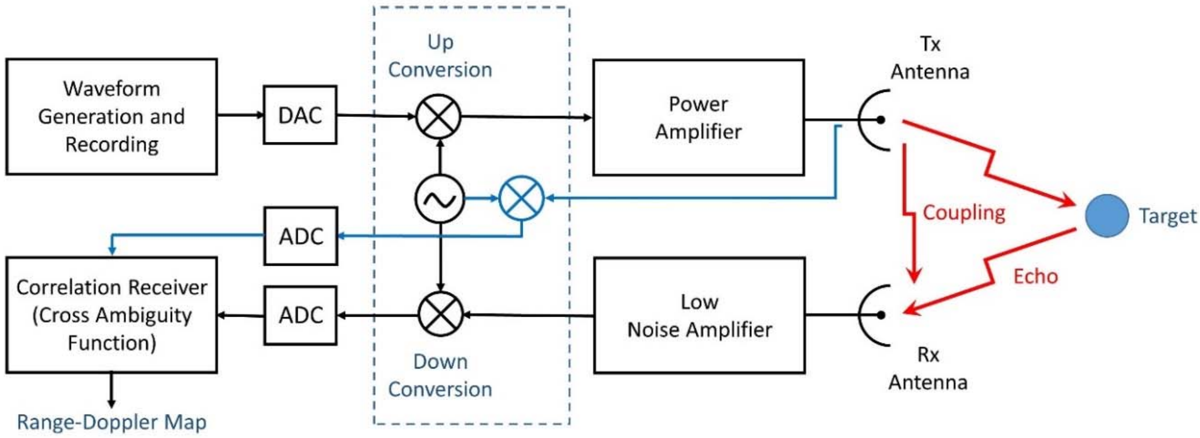


Figure 4. Basic Block Diagram of a Noise Radar. The reference is the record of the transmitted signal at the antenna port. ADC = Analog-to-Digital-Converter. DAC = Digital-to-Analog-Converter.

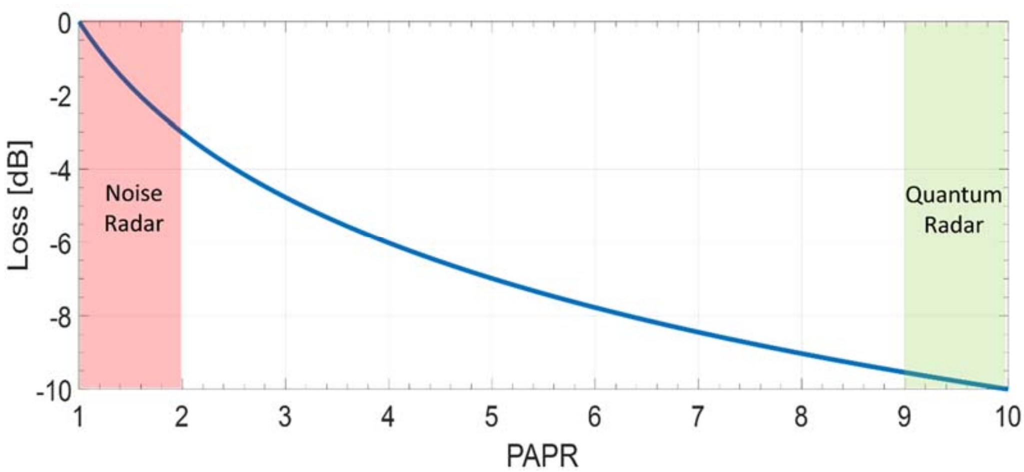


Figure 5. SNR loss (dB) versus $PAPR$, i.e. $Loss = -10 \cdot \log_{10}(PAPR)$.

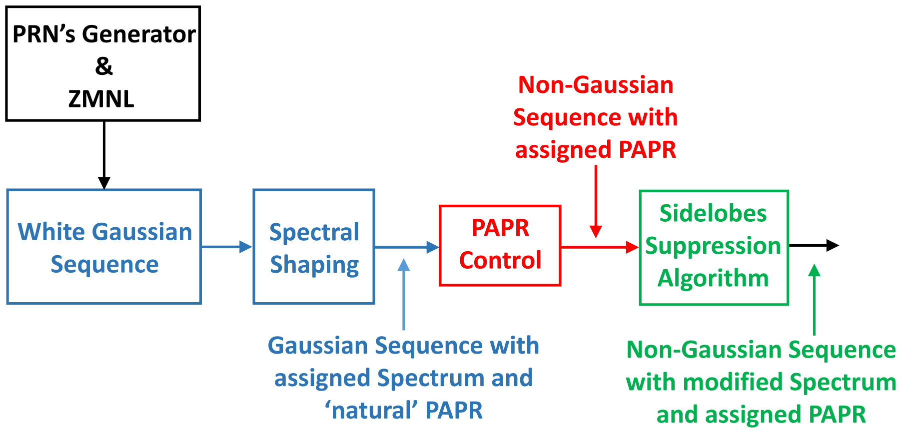


Figure 6. Basic Block Diagram of the waveform generator for a modern Noise Radar. PRN: Pseudo Random Number, ZMNL: Zero Memory Non Linearity. PAPR: Peak-to-Average Power Ratio.

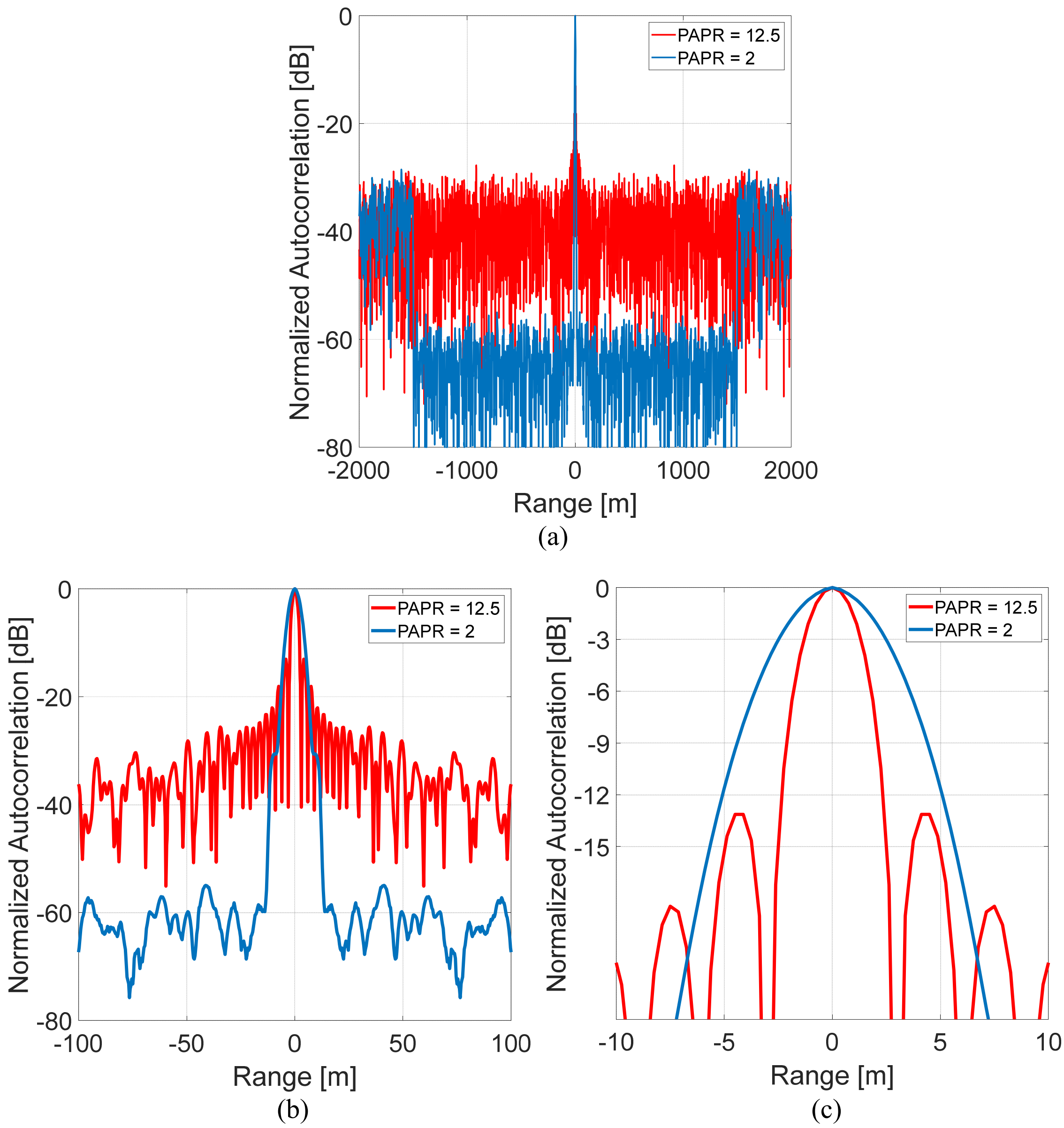


Figure 7. For NR signal: $B = 50$ MHz; $T = 100\ \mu s$ (i.e. $M = BT = 5000$); noise and interference absent; the NR waveform has been tailored to put the sidelobes of its autocorrelation function below -70 dB ($PSL \leq -70$ dB) in the delay region of $\pm 10\ \mu s$ (corresponding to $\pm 1500\ m$). (a) Output of the compression (matched) filter (blue line: NR, red line QR). (b) Zoom around the mainlobe, (c) Zoom of the mainlobe.

The difference between NR and QR is about $7 - 8\,dB$ as shown in Fig. 5. It largely compensates for alleged (and theoretical) maximum QR advantage of $6\,dB$.
Using *"tailored"* pseudorandom waveforms with respect to the *naturally Gaussian* QR waveforms (having PAPR$= 12.5$), a comparison between the outputs of the compression (matched) filter is shown for NR ($PAPR = 2.0$) and QR in Figs. 7a, b, c.

By these considerations, comparing NR and QR, it is clear that:

(i) NR may transmit every kind of pseudorandom signals: with constant modulus (phase-only code) or with a suited *"tailoring"* of the signal [42], i.e. allowing a PSL $<$ -60dB and acceptable loss related to PAPR, with more generally non-Gaussian statistics. In QR, the signal is *"naturally"* random and not modifiable (*PAPR* is around 9 - 10 as shown in green area of Fig. 5).

(ii) In NR the decision on target being present or absent is made on the basis of the output of the correlator (matched filtering). In QR, it is mostly based on a correlation coefficient (see the following section 4).

4. Historical Premises and Present Situation of Quantum Radar

4.1. Short History and Operation of Quantum Radar

Despite its appearance, the idea of *quantum radar* and of *quantum illumination* (QI) is not new; the history is resumed in [18], [19], [23], [43] and [44], with [19] including the optical regime (Lidar), the radar measurements and over one hundred References, and [49] including an up-to-date synthetic and honest presentation of the matter. In the introduction of [49] it is explained that, strictly speaking, the term Quantum Illumination (for short, QI) is more general than Quantum Radar even when the considered application is limited to microwaves. However, for the applications considered in this work, microwave quantum illumination systems and quantum radars are synonymous.
The early QI concept is found in two 2008 papers, [45] and [46], as well as – with a description of a possible embodiment – an old (2008) USA patent (assignee: Lockheed Martin Corporation) [47]. Incidentally, it is worth remarking that in the last fifteen years Lockheed Martin, to the best of our knowledge, has not produced more patents on QR nor has developed or anyhow exploited this early invention.
The patent [47], in its paragraph 5.1.3, describes the QI operation: *"…transmits quantum signal states of light that are entangled with quantum ancilla states of light that are kept at the transmitter"* and Fig. 5.2 of the patent shows *"an entangled pair of photons, one stored in the transmitter and another reflected from the target, sent to a measuring device"*.
About the situation in the period 2008 – 2020, from [49] we quote: *"Indeed, there has been a great amount of fanfare within the defence community and particularly the media concerning military applications of quantum radars."*
Most research on Quantum Radar is based upon the QI concept, within the obvious, understanding of the fundamental fact that the *"reflected"* photon, having interacted (at least) with an object, i.e. with the target, loses the entanglement.
The QI operation starts with a generation of pairs of entangled photons, the *idler* and the *signal* photon. The signal photon is sent to region where a target could be present, while the idler is stored. If a target is present, the radar may receive the signal photon after the transmission delay, otherwise the radar only receives noise photons. The QI protocol can be shortly described in four steps.

(1) A signal-idler photon pair is prepared M times, with $M \gg 1$.

(2) The idlers are coherently recorded.

(3) The signals, each with a mean (statistical average) number N_s of photons (with $N_s < 1$), are transmitted – each one in a pulse of duration $1/B$, i.e. the inverse of the system

bandwidth B – to probe at a given distance where there may be a target. The total radar time T for each Range interval equals M/B i.e. M is limited by the bandwidth-time product BT whose order of magnitude may be from millions to hundreds of millions.

(4) After the delay-time of the assumed target distance, the receiving sub-system gets either just noise (target absent), or the reflected signal plus noise (target present).

The presence of the target is statistically decided by correlating the signal with the idler. Theoretically, as shown in [46] the QI protocol should obtain an enhancement of $6\ dB$ in the SNR, equivalent to an error probability (assuming equally likely the presence or the absence of the target) four times smaller than a benchmark classical strategy. However, this $6\ dB$ advantage requires optimal global quantum measurements among all M pairs, which is well beyond any present or a foreseeable technology. The theoretical *quantum advantage* reduces to $3\ dB$ by joint measurements on individual mode pairs with local operations and classical communications. This $3\ dB$ advantage has never been attained in practice because of the impairments of real equipment.

The present QR literature assumes a single target to be either present (50%) or absent (50%) at a given point in the illuminated path. This is very different from real radar operation. In fact, in addition to the basic detection function, a radar, by definition, is requested to measure the *distance* of the targets, i.e. the Ranging. This capability is a fundamental one and can be exploited with an outstanding accuracy, often sub metrical, which distinguish radar from most sensors.

Ranging is not trivial when using the quantum illumination protocol, as quoted in [48]: *"… hinging on a joint-measurement between a returning signal and its retained idler, an unknown return time makes a Quantum Illumination-based protocol difficult to realise"*.

To all quantum radar protocols, the Range measurement is probably the most significant problem, which, added to the energy problem (i.e. the extent of two-way propagation losses for far targets) which is a focus of this work, prevents the realization of a real operating quantum radar.

The References of [18] include some theoretical solutions for the range problem. Another relevant problem for the QI is the one of the idler storage, which should be lossless throughout the signal's round-trip time to effectively correlate with the return signal. However, keeping unmodified the reference signal until the arrival of the corresponding echo signal is difficult, especially at radar (microwave) frequencies.

The readers interested to get an updated and comprehensive overview of the research on QR may refer to [23] and to [49].

4.2. Quantum Radar Today - Quantum Advantage - Stealth Targets

In addition to the *interferometric quantum radar* referenced in [25], two main classes of quantum radars have been proposed in the open literature: *quantum illumination* (QI) radar [46] and *quantum two-mode squeezing* (QTMS) radar [20].

Among the (not numerous) experimental evaluations, in 2020 Barzanjeh et. al. [50] carried out an experimental verification of Quantum Illumination in X-band, with generation and amplification of entangled microwave photons (frequencies: $10.09\ GHz$ and $6.8\ GHz$) in cryogenic conditions (at $7\ mK$) and with a target at room temperature and at a fixed distance of one metre. The experiments showed $1\ dB$ advantage over the optimal classical illumination at $N_S < 0.4$, the difference with respect to the theoretical $3\ dB$ being explained by the limitations due to the experimental set up.

Aimed to solve the ranging problem (as described at the end of the previous paragraph), the *quantum two-mode squeezing* (QTMS) protocol, which operates in a way closer to that of a conventional radar, has been implemented as a laboratory demonstrator (but with no target), [14], [20], [51]. It circumvents the ranging problem as follows. In the QTMS radar, the reference-entangled beam is immediately measured using heterodyne I (in-phase) and Q (quadrature) detection, while the received signal is measured at its arrival to be simultaneously correlated with the reference signal. This procedure requires maximally entangled pairs of photons; therefore, the process of spontaneous parametric down-conversion is the most used, generating a Gaussian two-mode squeezed-vacuum state at microwave frequencies.

Despite the loss of the entanglement due to the interaction with the environment (including the amplification), QTMS radar is aimed to exploit the correlation caused by the entanglement to detect the signal photons in noise when the correlation is computed many times. The number of pairs (or *"of pulses"*) M, i.e. the number of modes, is the time-bandwidth product: $M = B \cdot T$, where T is the duration of the emitted signal (less than, or equal to, the time-on-target) and B is its bandwidth. In each mode, an average number N_s of photons is transmitted; as for $N_s \gg 1$ the classical physics applies, while a quantum advantage is fully attained when $N_s \ll 1$.

Some recent technological developments, related to QR, include the wide-band Josephson Parametric Amplifier (JPA), a microwave resonant cavity terminated by a Superconducting Quantum Interference Device (SQUID) [12], [13] and the optical technologies, [52], [53]. A proposed general scheme of optical quantum radar [54] uses, in transmission, an electro-optical down-conversion, permitting to create the entangled photons in the optical region, and to down convert them to microwave photons; in reception, an up-conversion back to the optical region for detection. In [55] we find at Section II the description of the operation of a JPA and of the need to keep it very close to the *absolute zero temperature* (i.e. at a few - typically *seven-milli-Kelvins*) within a bulking dilution refrigerator. The latter is described in [55] as having the size of a large car including the He-3 and He-4 large Dewar's, with a power consumption alleged as large as 15 kW. Its high cost (order of 10^6 € according to [56]) causes a cost of the QR set (Fig. 1 of [56]) to be *five orders of magnitude* greater than the equivalent conventional radar.

In front of the significant *SWaP* (Size, Weight and Power) implicit in the QR technology, one may ask what radar performance enhancement arise from the Quantum approach.

The aforementioned theoretical evaluations show that the a gain depend on the quantum protocol, on the average number of signal photons N_s and of noise (background) photons N_B, sometimes called *"thermal" photons*, see section 5.1. An overview of the attained Quantum Advantage as at May, 2020 is found in [57] with a synthesis of experiments in Table 1 of [57], and in the more recent [23] where the Quantum Advantage is evaluated in detail and found, using TMSV and in the low SNR regime, less than 3 dB (Eq. (32) and Fig 3 of [23]). This is compared with the results by [58], [59], showing – for the first time – an experimented Quantum Advantage. It amounts to 1.2, i.e. about 0.8 dB, versus the 2.8 dB computed in [23] at equivalent conditions: the 2 dB difference is reasonably due to the imperfections in experimental set-up.

Note that in a conventional monostatic radar the link budget may be improved by 2.8 dB *(resp. 0.8 dB) increasing the antenna size* 1.38 *times (resp.* 1.1 *times) or increasing the transmitted power* 1.9 *times (resp.* 1.2 *times), with a much more limited SWaP than using the QR approach.*

The previous considerations and the ensuing evaluations show that the *"long-distance detection"* of QR in the title of [13] will not apply to real world situations, in spite of the known or anticipated technological improvements.

Of course, the same applies to the alleged detection of *stealth targets*, see [10] (Vella 2019) and [12]. In the Abstract of [12] we read: *"… making our MQI (Microwave Quantum Illumination) system a promising candidate for the detection of stealth objects"*. However, stealth targets call for a high power illumination, which is contrasting with the QR nature itself, as explained later on. Moreover, the literature (see [51], [60], [61], [62]) shows that the radar cross section of a target shall not significantly change passing from the Classical Radar (CR) to the QR, in similar operating conditions (in reality it does not change at all as clearly shown in [63]).

Simply said, the radar cross section of a target (either stealthy or not) does not change when QR is used, as the path of each photon to the target is not well defined because of the position uncertainty, and this causes some quantum interference which exactly replicates, in the far-field limit where the radar cross section is defined, the classical scattering behavior of electromagnetic waves.
In [14], [20], [51], [64] it is claimed that the above-described QTMS radar operation is similar to the one of a Noise Radar (NR) [38], [39]. In reality, NR and QR are quite different as one may see comparing Fig. 2 of [39] for the NR with Fig. 2 of [20] for QTMS radar.
In [20] a comparison is done with a particular (and quite *"artificial"*) classical radar demonstrator, named Two-Mode Noise (TMN) radar and implemented as close as possible to the QTMS radar demonstrator (including the cryostat refrigerator at a few mK), but with the pair of signals generated by mixing Gaussian noise with a sinusoidal carrier, i.e. not being entangled.
In some experiments, when both systems transmit in the air (from a horn antenna to a close-by similar antenna) signals at $-82\ dBm$, it resulted that the QTMS radar required one eighth of the integrated samples of the TMN radar to achieve the same performance in terms of Receiver Operating Characteristics (ROC curves). However, the Authors of [20] warn that a similarity between a NR and the TMN radar demonstrator stays only in the fact that both NR and TMN transmit random signals.
In this frame, it is very important to remark that the randomness in the TMN radar (and, more generally, in QR) is unavoidable and uncontrollable, being due to the quantum-mechanical signal generating process, while in modern NR they are digitally-generated and can be *tailored*. The consequences of these different generation processes of QR and NR include a poorer performance of the QR, as discussed in Section 3.2.
The ensuing part of this work treats the detailed analysis of Range performance.

5. Maximum Range for Quantum Radar

5.1. Some simple photon–number analysis

The Quantum Radar operation is based on target detection by idler-signal photons correlation, as previously explained. On the other hand, detection of a radar target depend on the energy transmitted on it and by its back-scattering, quantified by the radar cross section (RCS). Let us consider a simple, cheap (about 7000 US$) X-band marine radar such as HALO 6 (https://panbo.com/how-simrad-halo-works-12-radars-in-one/). It is a solid-state set which transmits, at frequencies around 9.4 GHz, 2.5 mJ towards the target (i.e. a lower energy than a less-effective legacy marine radar with a 6 kW magnetron and 1 μs pulse, which transmits 6 mJ). Its Range is:

- 5.32 km for a target with a RCS of 1 m^2.
- 8.8 km for a small vessel with Corner Reflector ($RCS = 7.5\ m^2$).

The energy of a single microwave photon at 9.4 GHz is about $6.22 \cdot 10^{-24}\ J$, hence, in order to reach the requested 2.5 mJ, the radar shall send to the target a number of photons about:

$$N_s \cong 4 \cdot 10^{20}$$

which, with an attenuation (Fig. 9) of the order of $-160\ dB$ and a number of background photons (Fig 8) of the order of 10^3, still leaves some SNR margin for target detection. This N_s value is consistent, for example, with an illumination time of $T = 100\ \mu s$, a bandwidth of $B = 100\ MHz$ and a "number of modes" BT of 10^4.
However, a corresponding QR should operate (assuming $N_s = 0.4$) with a 10^{21} times larger BT product, which, for the same bandwidth, corresponds to $T =\ 10^{17}\ s$, that is, over three billion years. More detailed Range evaluations for CR and QR follow.

5.2. General Remarks on Bose-Einstein Statistics and Noise Background

Determining the maximum operational Range of a radar set seems feasible with a simple computation of the Radar Equation, historically standardized by the classical report (and the ensuing book) on Pulse Radar Range by L. V. Blake, [65] and by the Marcum's theory [66]. However, the task is more difficult when considering many factors affecting the computation (target fluctuations, equipment losses, multipath, internal and external disturbances and more), so much that the real operational radar Range sometimes is as short as half of the computed Range. This is a worst-case (mentioned by M. I. Skolnik, [67] but errors as large as 3 or 4 dB are probably very common.

The maximum Range of a QR is an even more complex matter because it is related to the statistical description of the involved photons. From the Bose-Einstein statistics (as applicable in the cases of *zero chemical potential*), the average number of photons per mode N_B (where the subscript B stands for background) versus the frequency (f) at a system temperature (T_s, in Kelvin) is:

$$N_B = \frac{1}{exp\left(\frac{hf}{K_B T_s}\right) - 1} \tag{1}$$

with $K_B = 1.38065 \cdot 10^{-23}\ J/K$ the Boltzmann's constant and $h = 6.62607 \cdot 10^{-34}\ J \cdot s$ the Planck's constant. At radio and microwave frequencies and at room temperature $hf \ll K_B T_s$, hence, considering the linear approximation of $exp\left(\frac{hf}{K_B T_s}\right) = 1 + \frac{hf}{K_B T_s}$, Eq (1) becomes:

$$N_B \cong \frac{K_B T_s}{hf} \tag{2}$$

Fig. 8 shows Eq. (1) varying f from $0.1\ GHz$ to $100\ THz$. The blue area ($N_s < 0.5$) is the optimal for Quantum operation (the QR system has the optimum Quantum Advantage for an average photon number $\ll 1$ and loses the advantage for a number of photons greater than about five). Dashed lines represent Eq. (2), i.e. the linear approximation.

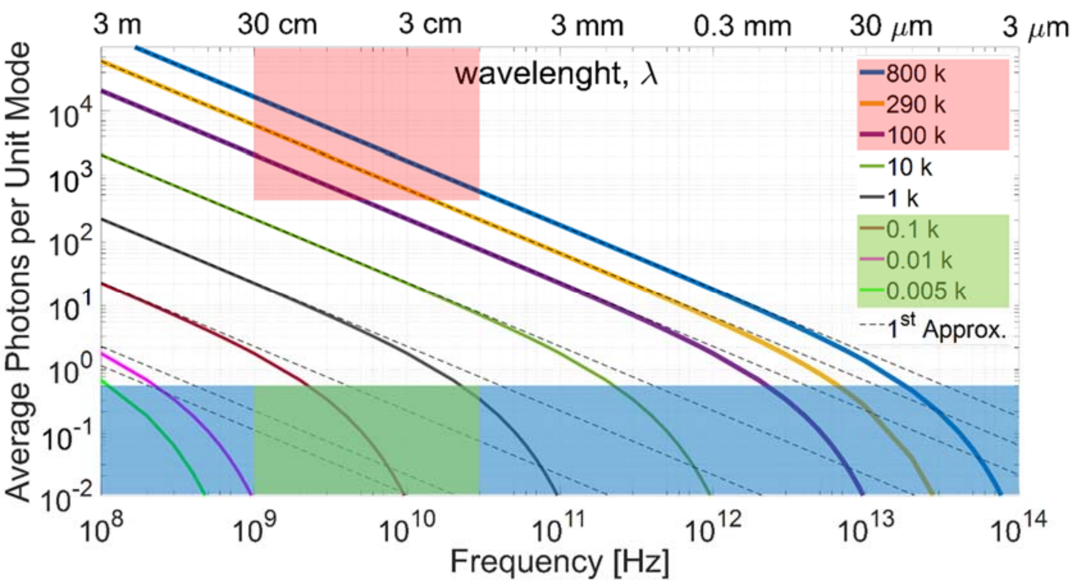


Figure 8. Average photons N_B per unit mode, Eq. (1), versus the frequency (up to 100 THz). $T_s = 0.005, 0.01,\ 0.1,\ 1.0, 10, 100, 290, 800\ K$. Dashed lines show the linear approximation of Eq. (2). The pink area represents the thermal background; the green area represents the signal photons.

The green subset of the blue area represents the average generated photons for a microwave ($1\ GHz < f < 300\ GHz$) QR, at temperatures below $1\ K$; the upper pink area represents the

much more numerous background photons from a favourable case of $T_s = 100\ K$ to a more frequent radar situation of $T_s = 290\ K$. A *worst case* of $T_s = 800\ K$ is also shown.
For an X-band ($\lambda = 3\ cm$) radar at room temperature N_B is of the order of 10^3, and is less than the unit only at T_s well below $1\ K$.

5.3. Range computations for Quantum Radar in the literature - Comments

The literature on the Range performance of a QR, often, simplifies the disturbance as *"background noise"* with N_B photons, while radar engineers know very well that the radar disturbance is something more complex. It includes unwanted echoes, propagation effects (with related attenuation), antenna noise, radiofrequency connections to the receiver, and finally, the first active reception stages, [65], [67] and [68].

In [69] we read (QI stands for Quantum Illumination, $\hbar = \frac{h}{2\pi}$, W is the bandwidth): *"In QI radar, the noise power of the receiver can be expressed as* $P_{rn} = \frac{\hbar\omega W}{exp(\hbar\omega/K_B T)-1}$, where K_B *is the Boltzmann constant and* T $(= T_s)$ *is the equivalent noise temperature in Kelvin. In practice, it is difficult to accurately calculate the noise temperature, which is related to factors such as antenna geometry, beam direction, solar activity, and signal wavelength etc. Therefore, in the subsequent analysis, we define the equivalent noise temperature of* $3 - 300\ K$, *which can represent the majority of radar operating scenarios".*

The lower value is quite optimistic: the system noise temperature T_s of a radar set is typically close to, or above, $250 - 300\ K$. In fact, the system noise temperature T_s [68], referenced to the output of the radar antenna, is the sum of three contributions: by the antenna T_a, the radiofrequency (T_{RF}) connections of the antenna with the receiver (including the duplexer and the rotary joint - if any) and, finally, by the receiver itself, whose main element is normally a Low Noise Amplifier, LNA (T_{LNA}). Hence, it results:

$$T_s = T_a + T_{RF} + T_{LNA} \tag{3}$$

with:

- T_a resulting from the external noise sources including: the *cosmic microwave background* (CMB), the sun, the stars, the atmosphere, the land and sea surfaces. In the absence of any radio stars in the antenna beam, the CMB assumes a minimum level around 5 K (see Fig. 8.19 page 526, chapter on Propagation of Radar Waves, of [67]), whose 2.7 K blackbody term uniform in all directions, is due to the radiation left over from the hot big-bang. Hence, T_a is a highly variable quantity. Common graphs [65] supply this contribution versus the operating frequency for a *"standard environment"* and for different values of the pointing angle θ of the antenna with respect to the vertical. For example, in the X-band (9 GHz), T_a has a maximum value of about $100\ K$ when pointing towards the horizon ($\theta = 90$ degrees) and a minimum value of about $10\ K$ in the unrealistic case of zenith pointing ($\theta = 0$). To set exemplary values, assuming $\theta = 30$ degrees, we have $T_a = 30\ K$ but for a Surface Movements Radar (SMR), whose antenna points down, T_a is close to the land temperature.
- $T_{RF} = (L_{RF} - 1)T_0$ being T_0 the reference temperature of 290 K (according to the IEEE standard) or, if known, the physical temperature of the previously mentioned RF connections, and L_{RF} is their attenuation (i.e. the loss). An exemplary value (for a 0.5 dB loss) is $T_{RF} = 35\ K$.
- $T_{LNA} = (F - 1)L_{RF}T_0$ where F is the noise figure of the amplifier. For an exemplary $F = 1\ dB$ and the above 0.5 dB loss, $T_{LNA} = 190\ K$.

The sum of all contributions yields a typically value $T_s = 255\ K$ (but over twice for a SMR).

In [43] it is explained that: *"… despite loss and noise that destroy its initial entanglement, quantum illumination does offer a target-detection performance improvement over a classical* *radar of the same transmitted energy. A realistic assessment of that improvement's utility, however, shows that its value is severely limited"*.

Basic and contrasting facts dominate the power budget, hence the computation of QR Range:

a) The energy in a single photon at microwave frequencies is extremely small when compared to the one of a Conventional Radar (CR) pulse: therefore, being the number of transmission modes (M), defined by operational constraints, one could try to increase the number N_s of average signal photons per mode. This increase brings back the radar system to the *classical operation*. With a number of photons per mode, say 0.01, optimal for quantum-advantage, the transmitted energy per microwave radar pulse (i.e. per mode), as shown in the example at Section 5.1, is 16 to 20 orders of magnitude below what is required for target detection. Moreover, according to quantum mechanics, the amplification of the radar signal generates noise, which would nullify the quantum advantage, [57].

b) Theoretically, a Quantum Illumination system shall provide a factor-of-four (6 dB) improvement in the error-probability exponent, i.e. in SNR, over its classical counterpart having the same transmitted energy [57], [64]. However, the considered classical benchmark was, in reality, a suboptimal CR), [20]. More important, note that the practical QR implementations limit the quantum advantage to lower figures, order of $1\ dB$ to $3\ dB$ only, [49], [57], [70].

c) The energy of a photon is proportional to its frequency, calling for QR operating, for example, in the terahertz bands, where, unfortunately, the atmospheric attenuation and the atmospheric phenomena prevent long-Range operation. To reach important energies for single-photon sensing one could go outside the radar context and resort to optical (energies 10^5 times higher) or even above, up to very-high-energy gamma rays having a single photon energy up to $160\ \mu J$, enough to reach the $2.4\ mJ$ exemplified in Section 5.1 with a few photons.

d) The increase of the number of modes M for a (necessarily limited) available signal bandwidth B would generate an increase of the pulse duration. Values of T above some threshold (order of a few milliseconds to hundreds depending upon the type and dynamics of the particular target) would render the system prone to the effects of target scintillation and Doppler frequency, destroying the correlation with the stored replica, hence nullifying the quantum advantage. In fact, in over fifteen years of QR studies and proposals, it seems that these relevant, well-known radar target features have been seldom taken into account.

Summing up, we quote from the Introduction of [70]: *"… while realizable experimentally, useful application of microwave quantum radar protocols to any conventional setting is unrealistic because of fundamental restrictions on power levels"*. Similar, honest conclusions are found in the most recent (2022-2024) literature.

The use of QR has been recently proposed for biomedical sensing [55], i.e. a short range (order of meters or less) case. Short-range radars have been introduced for healthcare applications of detecting human vital signs as early as in 1975. Heart rate measurements have been successfully measured at $1\ m$ distance using sub-micro-watt power levels, [71], and FMCW radar technology. Non-invasive microwave techniques for remote sensing of respiratory and circulatory movements have been developed with the average power density of radiated energy ranging from $1\ \mu W$ to $1\ mW$ per square cm, i.e. much lower than the ones due to the cellular phones used by patients and sanitary personnel. Some systems are capable of measuring heart rate and breathing rate at distances of $1\ m$, or behind thick layers of non-conductive walls.

In healthcare applications, the QR is not a useful option, because: (i) the SWaP limitations are obviously important (contrary to what is written in [55]); (ii) the short distances imply very low transmitted microwave power levels for which a $6\ dB$ (or lower) advantage is immaterial.

5.4. Exemplary Range computations for Quantum Radar

Some computations for a representative QR are shown in the following to sustain the previous discussion. The related main parameters are:

- f_0: operating (central) frequency;
- $\lambda = c/f_0$: wavelength;
- B: operation bandwidth, i.e. $f_0 - \frac{B}{2} \le f \le f_0 + \frac{B}{2}$;
- T: signal duration (less or equal to the dwell-time);
- $M = B \cdot T$: number of modes;
- σ: radar cross section (RCS) of the target;
- G: antenna gain (the same for Tx and Rx);
- T_s: system noise temperature;
- SNR: signal-to-noise ratio;
- L: total loss ($L < 1$);
- a_R: free-space two-way attenuation in the radar equation for a target at a distance R;
- η_Q: quantum advantage.

First, the free-space two- way attenuation is:

$$a_R = \frac{G^2\lambda^2}{(4\pi)^3 R^4}\sigma \tag{4}$$

In the QR/QI literature the attenuation is always indicated by κ, with a value arbitrarily set to $\kappa = 0.01$ i.e. $-20\ dB$. Eq. (4) is shown in Fig. 9 for a monostatic X-band radar with $1\ m^2$ of RCS. At a distance of $1\ km$, the attenuation is $-123\ dB$ for $G = 30\ dB$.

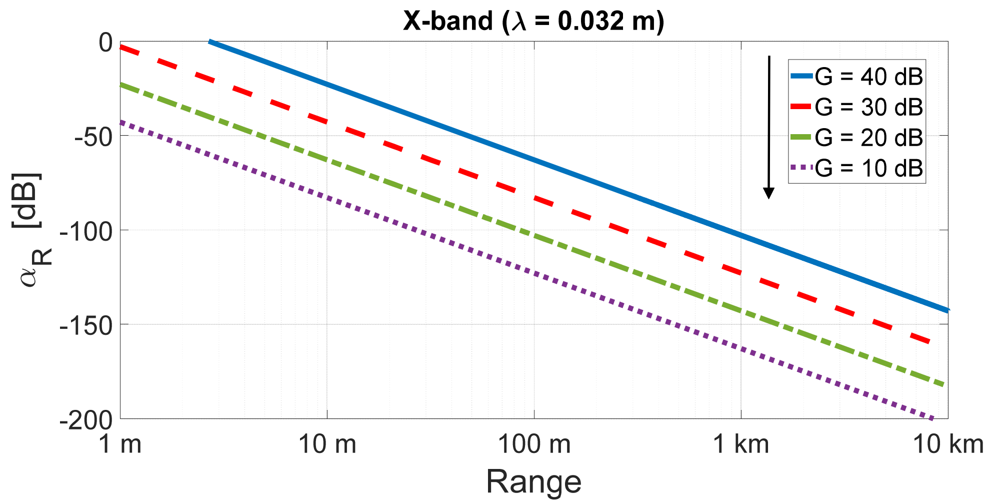


Figure 9. A Free-space attenuation a_R [dB], versus Range at X-band ($\lambda = 0.032\ m$), with $\sigma = 1\ m^2$ and $G = 10, 20, 30, 40\ dB$.

The received power from the target at a distance R is:

$$P_{rs} = M \cdot N_s h f_0 B \cdot \eta_Q \cdot a_R \tag{5}$$

where N_s is the average number of photons per mode and η_Q the quantum advantage.

From Eq. (2) the background noise power in a bandwidth B around f_0, is equal to the classical relationship:

$$P_{rn} = K_B T_s B \cong N_B h f_0 B \tag{6}$$

Hence, using Eq. (5) and Eq. (6), the signal-to-noise ratio is:

$$SNR = \frac{P_{rs}}{P_{rn}} = M \frac{N_s \cdot \eta_Q}{N_B} \cdot a_R = B \cdot T \frac{N_s \cdot \eta_Q}{N_B} \cdot a_R \quad (7)$$

To achieve a *positive* (in dB) SNR at range R, the Quantum Radar shall operate with a time duration:

$$T \geq \frac{N_B}{N_s \cdot \eta_Q} \cdot \frac{1}{B \cdot a_R} \quad (8)$$

At X-band ($f_0 = 9.4\ GHz$) assuming a bandwidth $B = 1\ GHz$ (about 10 % of f_0), for the sake of simplicity setting $N_s \cdot \eta_Q = 1$ (e.g. we assume $N_s = 0.25$ with $\eta_Q = 6\ dB$ or $N_s = 0.5$ with $\eta_Q = 3\ dB$), $\sigma = 1\ m^2$, $R = 1\ km$, an antenna gain (the same in Tx and Rx) $G = 30\ dB$, with $T_s = 290\ K$, it results $T \geq 346\ h$ which appears absurd. With a target much closer, i.e. at $R = 10\ m$, the SNR is 10^8 times greater than at 1 km and an operation with same BT should permit T to be at the order of *ten* milliseconds. From Eq. (8) we can reduce the time increasing B, however, the use of the electromagnetic spectrum by radar is regulated by the ITU (an ONU agency) and the general radar bandwidth allocation does not exceed 10 %. Therefore, the theoretical possibility of radar transmissions in an ultra-wide band is limited in principle to indoor, very short-Range, applications [72]. More exemplary time values are shown in Fig. 10 for $T_s = 100\ K, 5.0\ K$ and $2.7\ K$.

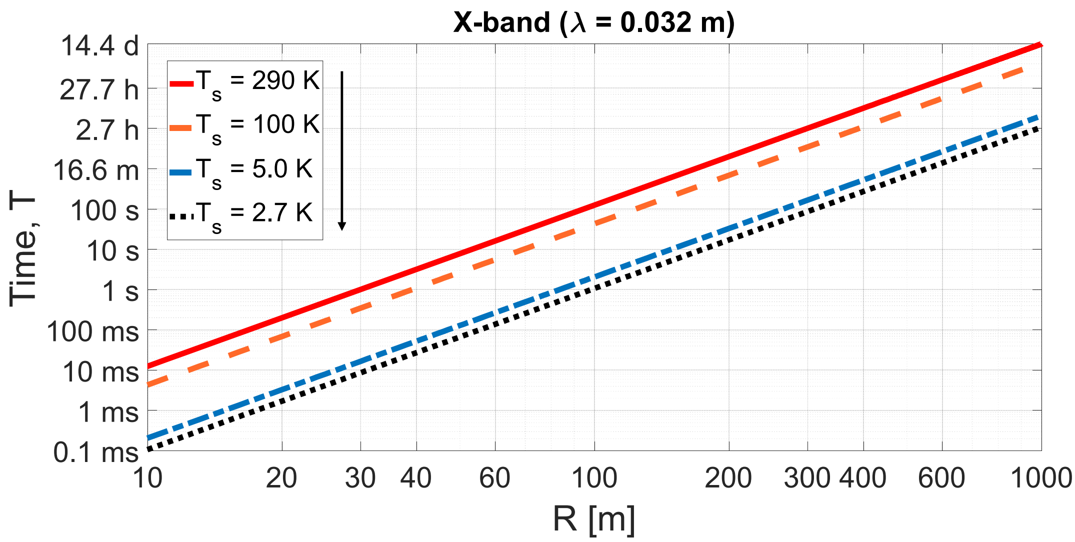


Figure 10. Minimum dwell time T to guarantee $SNR > 0\ dB$ versus Range (m) at X-band ($f_0 = 9.37\ GHz$) with $B = 1\ GHz$, $N_s \cdot \eta_Q = 1$, $\sigma = 1\ m^2$ and $G = 30\ dB$, $T_s = 100\ K, 5.0\ K$ and $2.7\ K$. Legenda: d = days; h = hours; m = minutes; s = seconds; ms = milliseconds. No atmospheric attenuation.

From Eq. (7) and Eq. (5) the maximum range (R_{max}) can be evaluated as:

$$R_{max} = \left[\frac{G^2 \lambda^2 L \cdot \sigma}{(4\pi)^3 SNR_{min}} \cdot \frac{N_s \cdot \eta_Q}{N_B} B \cdot T\right]^{\frac{1}{4}} \quad (9)$$

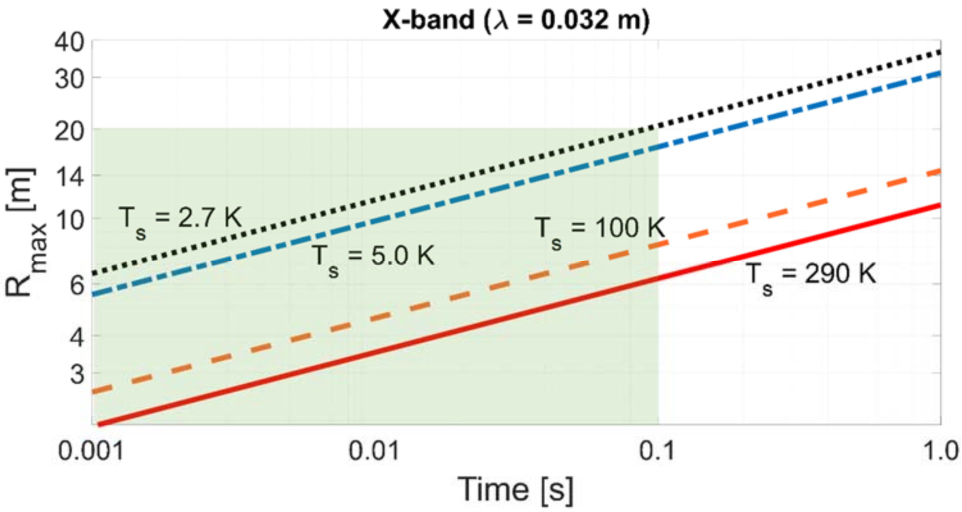
where L is the total loss, neglecting the attenuation of the medium.

Setting: $G = 30\ dB$, $L = -4\ dB$, $SNR_{min} = 13.2\ dB$ [66], $N_s \cdot \eta_Q = 1$, $\sigma = 1\ m^2$, $\lambda = 0.032\ m$, $B = 1\ GHz$, varying the time duration T from 1 ms to 1 s (i.e. M from 10^6 to 10^8), Fig. 11a shows the maximum range versus the time duration.

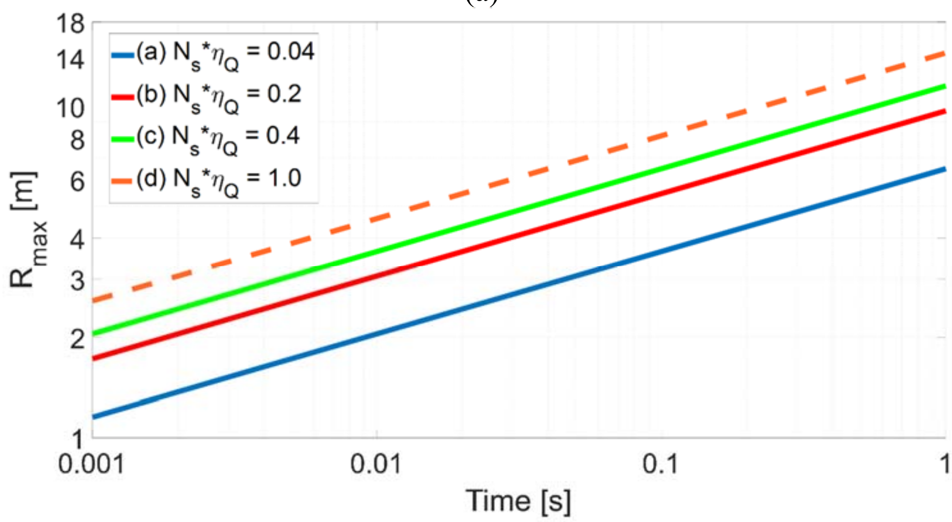
For $100\ K < T_s < 290\ K$, R_{max} is less than 10 m. Considering the CMB contribution around 5 K or 2.7 K, the maximum Range is less than 20 m with $T < 0.1$ s. Ranges greater than about ten or twenty meters can only be achieved when the whole radar set, including the antenna (and

the external surfaces within its main lobe) is cooled at cryogenic temperatures, which is not compatible with any use outside a specific laboratory.
The above evaluations used the simplified and conservative condition $N_s \cdot \eta_Q = 1$ to define the order of magnitude of the QR Range. More precise evaluations are shown in Fig. 11b for $T_s = 100\ K$ and the following cases and values:

	N_s	η_Q	$N_s \cdot \eta_Q$
a. QR – theoretical η_Q – low N_s	0.01	4 (6 dB)	0.04
b. QR – potential η_Q	0.1	2 (3 dB)	0.2
c. QR – theoretical η_Q	0.1	4 (6 dB)	0.4
d. QR – optimistic η_Q – high N_s	0.66	1.5 (1.76 dB)	1.0



(a)



(b)

Figure 11. Radar range, R_{max}, of an X-band QR vs time-duration $T\ [s]$ at $B = 1\ GHz$, system losses $L = -4\ dB, G = 30\ dB$. (a) $N_s \cdot \eta_Q = 1$ with $T_s = 2.7, 5.0, 100, 290\ K$. (b) $T_s = 100\ K$ with $N_s \cdot \eta_Q = 0.04, 0.2, 0.4, 1.0$. No atmospheric attenuation, no clutter, no radiofrequency interference. The typical interval for most radar applications is evidentiated in green.

In order to stay independent of the radar operating wavelength λ (eliminating the λ^2 term in the radar equation), in [23] they take, as a standard reference target, a trihedral triangular Corner Reflector (TCR) with side a, i.e. with RCS: $\sigma = \frac{4\pi a^4}{3\lambda^2}$. The considered target is an object with size $A > a$. Note that at the typical X-band radar wavelength of 3.2 cm, it results $\sigma = 1\ m^2$ for a relatively small TCR with $a = 0.125\ m$, while for $a = 1\ m$, the resulting X-band RCS is quite large, i.e. $4000\ m^2$. From Fig. 6 of [23], and for an object size of $a = 1\ m$ (corresponding to 36 $\frac{dB}{m^2}$ of RCS for the TRC in X-band), in the "feasible scenario" realm of [23] , the radar Range is between 10 m and 40 m, i.e. of the same order of the evaluations in this work.

Note that the situation improves with the frequency increases (and N_B quickly decreases, see Fig. 8), but above *circa* 8 GHz the attenuation by rain becomes a very critical factor and, with increasing frequency above 35 GHz, the attenuation by the atmosphere also becomes critical, preventing QR for any operational *"outdoor"* either civilian or military radar application.

In general, such an order of magnitude of the maximum Range is similar to the one shown in Figs. 2a and 3a of [69] evaluated for $T_s = 300\ K$ and $T_s = 3\ K$. Note that the advantage of QR over the CR, as shown in these Figures, is negligible when $N_s > 1$ and that for $N_s < 1$ the QR Range does not exceed a few *tens* of meters at X-band. Different evaluations (at X-band again) are shown in [74] (see Fig. 3) where the system noise temperature T_s is set to 10 mK, which is only possible when the radar and the environment are closed in a cryogenic generator and shielded from the 2.7 K CMB. However, this situation is not compatible with any real use. Similar detection performance for a millimeter-wave QR at 95 GHz are shown in Fig. 12.

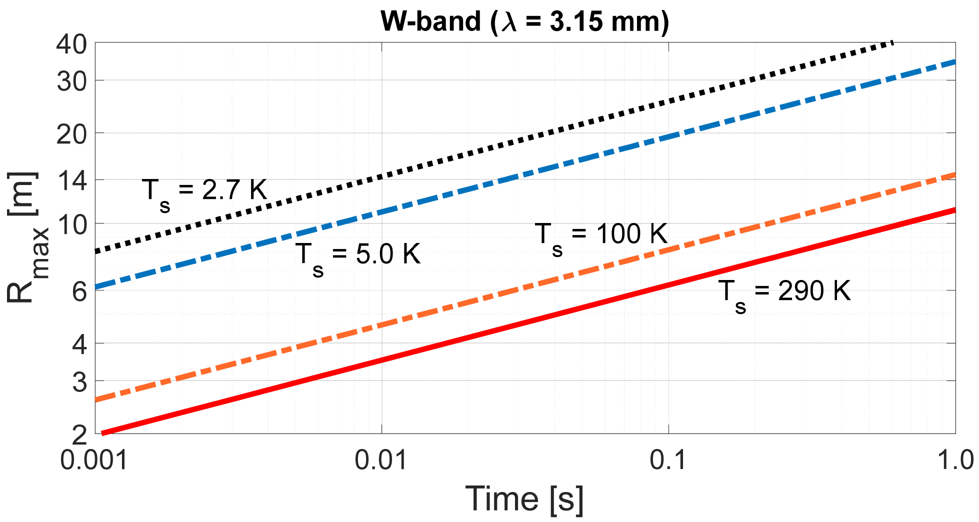


Figure 12. Radar range R_{max} of an W-band QR vs time-duration T at $B = 10\ GHz$, $N_s \cdot \eta_Q = 1$, system losses $L = -4\ dB$. No atmospheric attenuation, no clutter, no radiofrequency interference.

5.5. Maximum Range: comparison Quantum Radar vs Noise Radar

In some papers [18], [73], [74] and [75] Quantum Radar is proposed because of its Low Probability of Intercept (LPI) features due to the intrinsic randomness of its emission. Similar characteristics belong to Noise Radar [38], [39], [40] and [41]. Hence, it is interesting to compare the performance of NR and QR in equivalent system configurations.

A simple comparison figure is the ratio of maximum Ranges: $\frac{R_{NR}}{R_{QR}}$. For Quantum Radar R_{QR} is computed by Eq. (9), while for the Continuous-Emission Noise Radar [42], we have:

$$R_{NR} = \left[\frac{P_T G^2 \lambda^2 L \cdot G_{INT} \cdot \sigma}{(4\pi)^3 SNR_{min} K_B T_s B}\right]^{\frac{1}{4}} \quad (10)$$

where G_{INT} is the coherent integration gain, equal to the product $B \cdot T_{INT}$, with T_{INT} the coherent integration time (T in QR), equal or less than the dwell-time. Hence, the desired ratio is:

$$\frac{R_{NR}}{R_{QR}} = \left[\frac{P_T}{K_B T_s B} \cdot \frac{N_B}{N_s \cdot \eta_Q}\right]^{\frac{1}{4}} \quad (11)$$

Assuming $N_s \cdot \eta_Q = 1$ (a simple, rough approximation for the most frequent cases in which $0.2 < N_s < 1$ and taking into account that the antenna and the receiving parts operate close to a room temperature, i.e. $T_s \gg 1\ K$ and $N_B \cong \frac{K_B T_s}{hf}$ (see also Fig. 8), Eq. (11) becomes:

$$\frac{R_{NR}}{R_{QR}} = \left[\frac{P_T}{hf} \cdot \frac{1}{B}\right]^{\frac{1}{4}} = \left[\frac{E_T}{hf} \cdot \frac{1}{M}\right]^{\frac{1}{4}} = \left[\frac{N_T}{M}\right]^{\frac{1}{4}} \tag{12}$$

where $E_T (= P_T T)$ is the energy coherently transmitted on the target, $M = BT$ is the number of modes and N_T is the related number of photons transmitted by the NR: $N_T = \frac{P_T T}{hf}$.

For $P_T = 100\ mW, f = 10\ GHz$ it results: $N_T = 1.51 \cdot 10^{22}$. To get $R_{NR} = R_{QR}$ one has to set $M = N_T$, i.e. an unthinkable bandwidth $B = 1.51 \cdot 10^{13}\ GHz$, that could only be achieved operating at unrealistic carrier frequencies above $10^{23}\ Hz$, that is, in the Gamma rays region of the spectrum.

A similar evaluation is presented in [74] where, however, cooling of both CR and QR sets at $10\ mK$ is considered and the ratio between Eq. (12) and Eq. (14) of [74] leads to a Conventional Radar/Quantum Radar Range ratio equal to: $\frac{R_{CR}}{R_{QR}} = \left[\frac{1}{M}\right]^{\frac{1}{4}}$ (note that in [74], the number of modes is m in place of M) which is rather in agreement with the ratio of red and blue curves of its Fig. 3 (referred to X-band and to $m = 6$, which for the bandwidth $B = 2\ GHz$ corresponds to an unrealistic illumination time of mere $3\ ns$) but not with the evaluations shown in this report: likely, there are errors in the computations (called *"simulations"*) of [74], invalidating its conclusions and the sentence in the Abstract: *"It is shown that the detection range of quantum illumination radar is larger than that of classical radar…"*.

Likely, the pulse compression gain was not taken into account, but it is necessary to mention the more recent paper of the same Authors [75], where the compression gain of CR is considered and it is confirmed that the conditions $N_s \ll 1$, $N_B \gg 1$ and $M \gg 1$ maximize the advantage of Quantum Illumination but unavoidably lead to very short radar Ranges. The conclusions of [75] include: *"... although QI shows its advantages, this advantage is limited to the case of very weak transmitted signal power, so it may be a challenge for applying QI to radar remote detection"*.

From Eq. (12) one easily computes the frequency f^* making $R_{CR} = R_{QR}$. Posing $B = a \cdot f$, with $a < 1$ (fractional bandwidth), it results:

$$f^* = \sqrt{\frac{P_T}{h \cdot a}} \tag{13}$$

Fig. 13 shows Eq. (13), confirming that Quantum Sensing tends to become useful at very high frequencies (e.g. in the optical and infrared realms, and above) and at very low power levels.

For an analysis of QR operation above millimetre wave frequencies, the interested reader may see [69]. The pertaining computation of QR Range is at $\lambda = 1\ mm$. However, atmospheric attenuation is not taken into account, unlike the useful graph in [70].

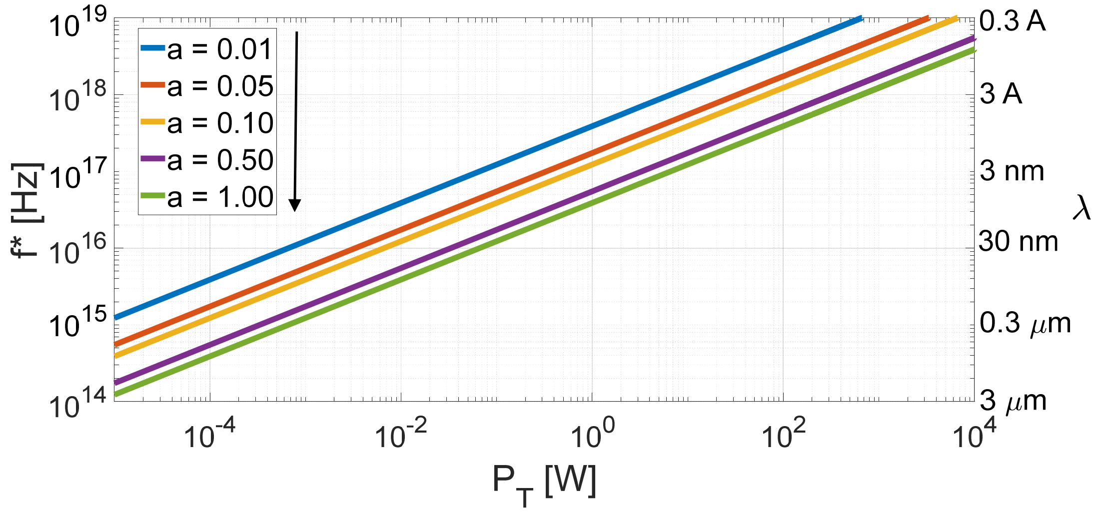


Figure 13. Frequency value f^* (and corresponding wavelenght) making $R_{CR} = R_{QR}$, Eq. (13). $B = a \cdot f$ with $a = 0.01, 0.05, 0.1, 0.5, 1.0$.

5.6. Specific Noise Radar's Advantage over Quantum Radar

Fig. 14 shows a comparison between the maximum ranges at the same operating conditions.

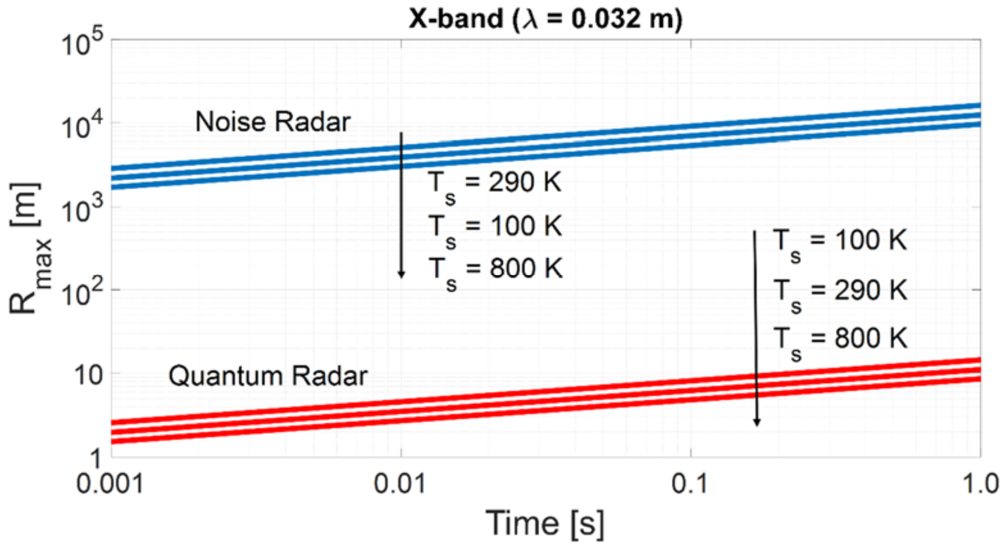


Figure 14. Comparison between the Maximum range for NR, Eq. (9), and QR, Eq. (8), at X-band vs time-duration T. Transmitted NR power: 100 mW (for 10 μW the lines are shifted below by a decade), and for both NR and QR: $G = 30\,dB$, $\sigma = 1\,m^2$, $L = -4\,dB$, $B = 1\,GHz$, $SNR_{min} = 13.2\,dB$, No atmospheric attenuation, no clutter, no radiofrequency interference.

For the Quantum Radar it is assumed $N_s \cdot \eta_Q = 1$, and the maximum range, Eq. (8), is the order of meters, while Noise Radar improves the Range, Eq. (9), to the order of ten kilometers (the system temperature is set to the realistic values of 100 and 290 K and to the exemplary *bad* value of 800 K).

6. Technical-Scientific Conclusions

The intrinsical Range limitations of a QR have been analyzed. Being this analysis based on the radar equation, which every graduated engineer knows well, one may wonder why these computations are absent in the QR literature before 2020, as well as in some literature after 2000. In fact, early Range computations are presented in [57] and in [70] and a criticism on practical use of QR is found in [56] and in [95]. Interestingly, radar equation is generally recalled in most QR papers and in Section 4.2 (Maximum Detection Range) of the early book

[25], where it is correctly written as Eq. (4.12) but without a definition of the *"Equivalent Temperature"* T_e. Most significantly, the radar equation is *never used* throughout the whole book. In more recent QR literature, Range computations are generally done for an unrealistic two-way propagation attenuation $\kappa = 0.01$ ($-20\ dB$) only, vs. the real-world $-100\ dB$ to $-150\ dB$, as shown in Fig. 9.

The quantitative analysis in this work was not extended to Range *measurement*, which is an unresolved issue in Quantum Radar. Other relevant considerations such as technical feasibility, operational problems and, last but not least, cost are found in [56] and [57].

A good synthesis on the operational problems of QR and its readiness is presented in [76] with numerous References. The conclusion of [76] includes what follows: *"Ultimately, one should not dwell on the – black-and-white what is better – mentality, but rather pursue this technology with the mind-set of intellectual curiosity and ignorance on its most appropriate application"*. This general approach would be acceptable, or even welcome, if applied to basic research. However, radar is an object of applied research, in which, for ethical reasons, we do not agree with promising impossible results to financing Bodies, especially when the impossibility may be shown with a few evaluations that could have been written on the back of an envelope since the beginning of the 2010's.

Summing up, there is no reason why a Quantum Radar, irrespective of the used protocol, shall perform better than a Conventional Radar. Therefore, some claims regarding stealth targets remain unmotivated and outside an operational perspective, see for instance [12]: *"Measurement results of the developed JTWPA, pumped at 12 GHz, show the capability to generate entangled modes in the X-band, making our Microwave Quantum Illumination system a promising candidate for the detection of stealth objects"*.

Finally, we conclude that in QR:

- There is a strong constraint on the transmitted power to get a limited (less than $3\ dB$) quantum advantage.
- The random nature of the transmitted signal does not permit any *"tailoring"*, resulting to a PAPR-related loss significantly greater that the quantum advantage.
- If a low-powered signal of a quantum noise radar is amplified, then a classical noise radar results, which outperforms the quantum radar.
- Quantum radars are much more difficult to implement than what some papers were claiming, and the work with signals photons in the microwave (or mm-wave) systems seems not a fruitful idea.
- The QR (more correctly, quantum illumination) concept might be useful for future quantum sensors in ultra violet, X-ray and Gamma domains. The related challenges consist in the lack of methods for efficient coherent signal processing at these frequencies, while conversion down to the frequencies below, say, 5 GHz, where ADCs are readily available, is also problematic. Moreover, at those frequencies the attenuation propagation and the target's back-scattering are much different than at microwave.

Conclusions similar to the ones of this work are appearing in widely-distributed journals such as Science [77] from which we report the following: *"Even if experimenters can overcome the technical hurdles, quantum radar would still suffer from a fatal weakness, researchers say. The entangled pulses of microwaves provide an advantage only when the broadcast pulses are extremely faint. The extra quantum correlations fade from prominence if pulses contain significantly more than one photon—which is overwhelmingly the case in real radar. 'If you crank up the power, you won't see any difference between the quantum and the classical', Barzanjeh says…."*.

Again in [77] it is noticed that the alleged military advantage of a quantum radar due to its covertness is practically immaterial due to its extremely short operating range.

For the military applications of QR, we quote from [24]:

"Quantum radar too is subject to the radar equation, where the received power is lost with the distance's fourth power. In parallel, to keep the quantum advantage, it is desirable to have one or fewer photons per mode. … This requires a lot of quantum signal generators, cryogenics, large antenna sizes, etc. All this leads to extremely high cost, and impractical design. … Apart from the high price, scepticism also remains about the detection of stealthy targets or jamming resistance. … In summary, the long-range surveillance quantum radar is unlikely to be achieved even as a long-term prospect".

The technical conclusions in this work, as applied to the defense, coincide with those of a study by Lincoln Laboratory/Massachusetts Institute of Technology (MIT) for the Under Secretary of Defense Research and Engineering of the USA, which states, in its executive summary, that [95]: *"Quantum radar does not have the potential for long range standoff sensing (>10 km) at radio frequencies (<100 GHz)"*. Supporting their conclusions are key findings that system requirements (namely superconductivity) needed to realize quantum enhancements are a limiting factor, and that the integration time for a returning pulse could require *up to three years of processing time*. The Defence Science Board (DSB), an independent Department of Defence (DOD) board of advisors, has also concluded that quantum radar *"will not provide upgraded capability to the DOD"*.

7. Concluding Remarks

7.1. General and Quantum Limits

Quantum-mechanical results are present in our daily life: we currently use relevant devices and applications such as transistor, laser and LED, Magnetic Resonance Imaging, atomic clocks and more. These applications, however, belong to the macroscopic and human world, while Quantum-Mechanics (QM) deals with extremely small elementary entities such as electrons or light photons: manipulating them is another matter. If QM were applied to macroscopic objects (e.g. well above the mass of an *alpha* particle), an attempt to solve its equations in the presence of a large number of elementary entities, such as for a mole of matter, will be so cumbersome as not to lead to usable results, and whenever successful, it will approximate very closely the classical solution.

Such a situation is similar to the one in *optics*. *Wave optics* (based on Maxwell's equations) is approximated very well by *geometric optics* in many practical cases. In fact, geometric optics is a particular case of solution of integrals using the approximated method of the *stationary phase*, which may work well because, in optics, the deviations due to wave effects are (very often) negligible when operating with the common optical elements such as lenses, mirrors, irises, …, which are much larger that the micrometric optical wavelengths.

Operating in the microwave region, the situation changes and the Maxwell's equations can be used, or sometimes refinements of the geometrical optics such as the GTD (Geometrical Theory of Diffraction), [78], [79].

In the quantum arena, an example is that of semi-classical (or semi-quantized) models in electromagnetism; see for instance [80].

Summing up, the fact that quantum theory (QT) in more complete than the classical one (CT) does not mean that QT has to be used everywhere, e.g. to define matters like an impossible *"Schrödinger's cat"* [81]. In the scientific literature, there are some attempts to use Quantum Mechanics in non-pertinent areas. A well-known one is by a Nobel Prize such as Sir Roger Penrose who for a long time – without any particular success – tried to explain the Consciousness applying Quantum Mechanics to the microtubules in the brain, see for instance [82], [83], [84] with the lack of success explained in [85]. A similar way, again related to the search of the nature of the Consciousness, has been recently followed by a respected former microelectronics leader, Federico Faggin [101].

For a critical approach to the inappropriate use of quantum mechanics in non-pertinent areas the interested reader is addressed to [86]. Finally, the explanation of quantum-to-classical transition in [21] deserves the final quote, which follows:
"... given that fundamental processes in the universe are quantum, ... the classical mechanics works because the macroscopic world has so many particles that the quantum states decohere and quantum mechanics becomes classic" […] *"the quantum-to-classical transition can be understood as a consequence of the mechanism of decoherence. Pure states remain pure only in closed systems"* [...] *"Even in the outer space the microwave background, if nothing else, acts to produce decoherence* [...]*".*

7.2. Orders of Magnitude and the Quantum Radar case

When considering the use of quantum mechanics, the *Order of Magnitude* (OoM) shall always be taken into account.

With the OoM in mind, let us consider a QR case. In [14] referring to the transmitted and to the reference signal of a radar, it is said that: *"… the Pearson correlation coefficient between two electromagnetic signals … because of the presence of quantum noise … cannot go to 1 unless the quantum noise in the two signals are correlated … arbitrary waveform generators cannot control the quantum noise in the signals… there is a limit to how well a signal and the record of that signal are correlated — a limit that can be overcome by using entangled signals …".* The reason for this is attributed to the well-known Heisenberg-Pauli-Weil *uncertainty principle* (see for instance [87], [88] and [89]) for the product of the standard deviations of two *conjugated variables* p and q (such as: position/momentum or energy/time) in the formulation by Earle Kennard:

$$\sigma_p \sigma_q > h/4\pi$$

where h is the Planck's constant. In [14] and [20] this idea is applied to quadrature components I and Q of the radar signal, stating that *"every electromagnetic signal must contain quantum noise. The correlation between any two signals will necessarily be limited as long as their quantum noise components are uncorrelated. This will always be the case for any pairs of signals created by methods based on classical physics"* and concluding in [20] that *"... according to classical electromagnetism, it is possible for two independently generated light waves to be perfectly correlated if they share the exact same waveform. This is impossible to realize in practice, of course: there will always be noise somewhere in the system. Astonishingly we find that, even in theory, the noise cannot be reduced to zero. It turns out that in-phase (I) and quadrature (Q) voltages of any signal cannot be simultaneously measured with infinite precision. In the appropriate units —* ***the specifics are unimportant*** *— they satisfy the inequality*:

$$\sigma_I^2 + \sigma_Q^2 \geq \frac{1}{4}$$

Quoting [90] (the applicability of this quote to this particular case has not been checked here) the Authors of [20] comment: *"This is an analogue of the famous Heisenberg uncertainty principle, but applied to quadrature components rather than position and momentum"*. Later they add: "$\mathrm{I}(\mathrm{t})$, *the time series of measured* I *voltages, is a random variable:* $\mathrm{I}(\mathrm{t}) \sim \mathrm{I}_{\mathrm{classical}}(\mathrm{t}) + \mathrm{N}(0, \sigma_{\mathrm{I}}^2)$*"* with $\mathrm{I}_{\mathrm{classical}}(\mathrm{t}) = \mathrm{A}\cos(\omega \mathrm{t})$ and, to be more convincing, they show in both papers [14] and [20] a drawing, plotted with arbitrary units (see Fig. 4 of [20]) showing a sine wave plus noise. The lack of the due consideration of the OoM is probably enhanced in most physics papers by the use of "convenient" or "natural" (i.e., not SI) units, in some cases [96] (Gaona-Reyes, et al. 2024) setting $h = 1$ or $\hbar = 1$ and/or $c = 1$ and/or $8\pi\mathcal{G} = 1$ where $\hbar$ is the Planck's constant divided by 2π, c the speed of light in vacuum and $\mathcal{G}$ the gravitation constant.

In reality, the specific **units are important** (as well as the OoM and the units)! When the actions are normalized to the Planck's constant, as in [90], the un-normalized I and Q, i.e. the one used by radar engineers, are easily computed. Taking the I component, its "quantum" variance, for $B = 1\ MHz$ and $T = 1\ ms$, is of the order of $10^{-25}\ V^2$, i.e. a power of the order of $-190\ dBm$, which is negligible in every real case (note that the noise in a radar receiver with $B = 1\ MHz$ is of the order of $-110\ dBm$ before amplification and the weakest useful signal is of the same order of magnitude).

A generated radar signal at $1\ V\ pp$ with an amplitude resolution of 16 bit has a minimum digitized level about $110\ dB$ above the $-190\ dB$ of the alleged *"quantum limit"*; even doubling the number of bits the *"quantum limit"* has no significant effect. The very low power levels in the generating section of entangled photons in the above-referenced QTMS radar demonstrator are of the order of *Attowatts*: the Josephson Parametric Amplifier (a resonant cavity with a superconducting quantum interference device (SQUID) at one end) has an output of $-145.43\ dBm$.

Hence, in [14] we read that *"… quantum entanglement (is) a potent weapon in the war of signal versus noise"* and that *"… modern radars … are rich in possibilities and capabilities, such as a synthetic aperture radar (SAR), adaptive array processing, and space-time adaptive processing. Can a quantum radar do all that? In the case of QTMS radar,* ***the answer is a definitive yes****"*. Remembering that SAR operates from airborne or satellite platforms at distances of the order of ten, hundred or thousand kilometres, **the answer** – based on the above discussed Quantum Radar equation and on the SWaP considerations – **is a definitive no**.

The lack of attention to OoM is found somewhere also when discussing the Noise Radar operation, in [91].

The correct Order of Magnitude is not always present in QR and QI papers. An example, among many, is found in [100], a 2023 theoretical work (not supported by real experiments) written by 5 authors from 6 international research institutions. In this paper, the range of parameters assumed for the evaluations is outside any practical sensing application. In fact, the number of modes (M) is of the order of 10^9 (see for instance Figs. 6 and 8) i.e. for a (reasonable) bandwidth of $1\ GHz$ the time-on-target is of the order of *one second*, too long with respect to the time constants of both radar air targets and biological phenomena such as heath beats and breathing. Moreover, the two-way attenuation is always set at $\kappa = 0.01$ which at X- band (10 GHz), for antenna gains of the order of 20 dB (a typical gain for X-band horns), and RCS of $1\ m^2$, corresponds of a radar Range of only 26 *cm*, which diminishes up to 8.2 *cm* for $10\ dB$ gain antennas (note that these figures shall be considered orders of magnitude only, as RCS and antenna gain are defined in the far field case). In spite of that, the Abstract of [100] concludes as follows: *"… the performance is further improved and also analyzed in terms of receiver operating characteristic curves. Our findings pave the way for the development of practical microwave quantum illumination systems"*.

7.3. The Common Sense and the need for Criticism

Engineering, including Radar Engineering, can be defined as the Science of Solving Problems as it finds solutions for real-world problems, not problems for allegedly found solutions; hence, it needs *Common Sense* and *Criticism*. Examples of lacking criticism (in particular, self-criticism) and common sense are found in many papers and books on QR.

a) Starting with the early book on Quantum Radar [25], an excerpt of its Conclusions (Chapter 7) – no comments are needed – follows. *"Quantum illumination offers higher signal-to-noise ratio which increases the detectability and identification of stealth targets even in the presence of noise or a jamming signal [67]. In addition, defining the quantum radar cross section using quantum electrodynamics leads to the result that the "effective visibility" of a target near the specular direction is increased if observed with a quantum radar instead* *of a classical radar [61, 62]. Furthermore, a quantum sidelobe structure offers the possibility of detecting RF stealth targets. As a consequence, quantum radars may be able to detect, identify, and resolve RF stealth platforms and weapons systems. In the electronic battlefield, quantum radar may become a revolutionary technology just as RF stealth technology was in the last three decades of the 20th century".*

b) From the same Author, in [97] we read: *"Space-based surveillance … represents a practical application of quantum sensing for the detection of near-earth objects which threaten spacecraft or terrestrial life"* and "*The proposed system consists of a set of spaceborne multispectrum quantum sensors operating in the optical and/or X bands to achieve super-resolution (beyond the Rayleigh diffraction limit) and super-sensitivity (beyond the shot noise limit) to provide a quadratic increase in detection sensitivity compared to classical alternatives".*
c) Again, in [99]: *"We compare the performance of quantum radar against a coherent light sensor (such as lidar) and classical radar. …. We show that, compared to the two classical standoff sensing devices, quantum radar is stealthier, more resilient to jamming, and more accurate for the detection of low reflectivity targets".*

The scientific community has recognized *"beyond reasonable doubt"* that QR is not suited to Stealth targets detection and the so-called *"quantum radar cross section"* is the same as the classical radar cross section, in agreement with the common sense. In spite of that, in 2024 still we find papers (presented at IEEE conferences) such as [102], where we read: *"Detection of Stealth Aircrafts - Stealth aircraft are hard to trace using traditional radar as it minimizes the reflections of the radio waves sent. Quantum Radars, however, will be much more efficient as they can detect subtle disturbances in the quantum states of photons, which will help in the detection of stealth aircraft".*

Referees, obviously not very active in this case (and others), should avoid these *"propagations of errors".*

More examples in the QR and stealth targets arena follow.

- Some papers promise detection capabilities of stealth targets, see for instance [11], where one reads: *"To detect a stealth target, one may directly probe it with a single photon* (sic !) *and analyze the reflected signals. The efficiency of such conventional detection scheme can potentially be enhanced by quantum illumination, where entanglement is exploited to break the classical limits …".*
- Some papers promise unrealistic results such as the microwave detection of objects under cloaking [92]: *"… a quantum illumination protocol to detect cloaked objects, and we have specifically studied the implementation in quantum microwave technology".*
- Some papers (e.g. [60], [61], [62]) describe theoretical analyses of an alleged *"Quantum radar cross section"* (QRCS) and found particular results which are demonstrated to be incorrect in [63]. Common sense would avoid useless efforts by the simple consideration that follows. When the radar target of a given radar cross section (RCS) has produced the backscatters phenomenon (whose entity in the far field is measured by the RCS itself) *it does no matter whether there are "ancilla" or "idler" photons somewhere or not. In other words, the target "cannot know" whether the photons impinging on it originate from a Quantum or from a Conventional radar*, as there is no way to distinguish both cases from the target point of view. Hence, common sense says that no difference may arise from QR and CR radar cross section, as confirmed by the ad-hoc detailed analysis in [63] and recalled in [21]. In spite of this very clear 2020 paper, some authors, neglecting or ignoring it, have continued to write on a non-existent QRCS, [109], [110], [111] and [112].

Summing up: sometimes we need to be suspicious about claims and results in scientific papers even if the author is a renowned researcher such as Kary B. Mullis, the inventor of PCR (Polymerase Chain Reaction), 1993 Nobel prize for Chemistry. Mullis wrote an ironical paper

[93], published by one of the most important scientific reviews, *Nature*, in spite of the funny nature of its content clarified since its abstract. After mentioning *Nature*, completeness concerning top Journals leads us to mention *Science*. A false result, the alleged polymeric structure of water or "*Polywater*", remained in *Science* for a decade probably because a missed use of common sense, see [94].

More recently, a high-temperature *"quantum night fever"* was possibly originating papers neglecting the OoM and showing allegedly indemonstrable claims such as [96] whose Abstract starts with *"Assuming that Quantum Mechanics is universal and that it can be applied over all scales, then the Universe is allowed to be in a quantum superposition of states"* and the Introduction follows with *"… if quantum theory is universal, then the Universe should be also quantum"*. The model proposed in [96] (using an arbitrary – stochastic and nonlinear – modification of the Schrödinger equation, i.e. of the linear and deterministic equation which constitutes the main pillar of quantum mechanics) is alleged as *indemonstrable* by the Authors themselves, as at page 8 we read: *"… one cannot distinguish a quantum Universe … as described by our model, from a classical Universe"*. From this paper and its *incipit* we learn that four researchers from seven organizations ignore (or, for some reason, deny) that quantum mechanics ***simply does not work at the macroscopic scale*** i.e., the scale of human's experience, an experience from which classical physics has been conceived during two millennia and tested. This is clearly commented in [21] at pages 284 and 265-266: *"…why does classical mechanics work at all? It works because the macroscopic world has so many interacting particles that the quantum states decohere and quantum mechanics becomes classic"*.

The scientific and technical reliability is not higher in patents, which are institutionally and legally checked only for three elements: a) Industrial application, b) Novelty, c) Inventive step.

In spite of the high cost of patenting, some Companies do no check the scientific/technical content and the basic feasability of the invention. An example is [47] where one reads (boldface added): "*An entangled quantum particle generator generates a signal including a plurality of entangled particles.* ***The wavelength of the signal is the sum of the wavelengths of the entangled particles"***.

In another patent with a similar title, namely US 2011 0026614A1 by the same inventors (Edward H. Allen and Markos Karageorgis) one reads: "*As used herein, the term "quantum sensor system" refers to systems that use entangled beams at radio and/or audio frequencies. The terms "quantum" and "quanta" refer to photon(s) for radio frequency waves and/or phonon(s) for audio frequency waves.* ***The wavelength of two or more entangled quanta, referred to as a multiquanta, is proportional to the number of entangled quanta associated with the multiquanta"***.

7.4. Quantum Radar History - Revisited

The history of Quantum Radar research can be divided into four periods – partly overlapped in time: (a) *proposals*, period: 2008 - 2011; (b) *early papers production*, period: 2011 - 2017; (c) *mass papers production*, period: 2018 - 2021; (d) *Disillusionment and endurance*: period 2022 - 2025; see also Fig. 2.

a) Early proposals, most rather generic, are mostly found in early writings in 2008-2011, whose forerunner and prototype is [25] (mentioned in Section 1.2), which deserves some more comments. In 166 pages, this book aims to treat the following: Introduction to Quantum Radar, The Photon (Maxwell Equations, Scattering, Radar Cross Section, Electromagnetic Quantum Fields…), Classical Radar Theory, Quantum Radar Theory, and finally, Quantum Radar Cross Section. The Eq. (4.7) is the standard radar equation, called *"the most fundamental equation in classical radar theory"*, and Eq. (6.25), Quantum Radar equation, only differs from Eq. (4.7) by the use of the alleged *"Quantum Radar Cross Section"* in place of the radar cross section, and by the (unexpected) absence of the attenuation term F.

A comparison of the results by both equations is absent throughout the book, so, one wonders why they are reported. However, the book does not lack advertising since the Introduction: *"As such, quantum radar offers the possibility of detecting, identifying and resolving stealth targets …"* and *"the effective visibility of certain targets is increased if observed with a quantum radar instead of a classical radar"*. *"Preliminary results show that quantum radar using entangled photons can provide a quadratic increase of resolution over non-entangled photons"*. In Section 1.3: *"Quantum radar offers the prospect of detecting, identifying, and resolving RF stealth platforms and weapons systems. In the electronic battlefield, quantum radar may become a revolutionary technology just as RF stealth technology was during the last three decades of the 20th century"*. Also: *"a quantum sidelobe structure offers a new channel for the detection of RF stealth targets"*. At the end of the Foreword: *"As we will see, quantum information technology may be the key to improved long range sensor systems"*. The exam of one Reference concludes the present (and short) analysis of the book. Reference [2] of [25] is the afore-commented patent application [47] by E. H. Allen and M. Karageorgis. Comments are left to the reader.

More works by Lanzagorta include [97]: *"Space-based surveillance ideally satisfies these conditions and represents a practical application of quantum sensing for the detection of near-earth objects which threaten spacecraft or terrestrial life"* and *"The proposed system consists of a set of spaceborne multispectrum quantum sensors operating in the optical and/or X bands to achieve super-resolution (beyond the Rayleigh diffraction limit) and super-sensitivity (beyond the shot noise limit) to provide a quadratic increase in detection sensitivity compared to classical alternatives"... "We described a basic system of orbiting quantum sensors which could be used for a variety of missions such as missile defense, Earth defense against comets and asteroids, and debris tracking for the safe removal and passage of spacecrafts. Due to the nature of quantum sensing, these satellite network system would have reduced cost and higher performance than what can be achieved with today's technology"*. No mention is found on the Cosmic Microwave Background nor on the Range Measurement!

Further Publications by Lanzagorta include [98] and [99]. From the *Abstract* of [99]: *"… we analyze the theoretical performance of low-brightness quantum radar that uses entangled photon states. […] We show that, compared to the two classical standoff sensing devices, quantum radar is stealthier, more resilient to jamming, and more accurate for the detection of low reflectivity targets"*.

b) Most of *early papers production* started after Lanzagorta's book; as in the book, experimental data are practically absent as well as any quantitative evaluations of QR Range by the radar equation.

   This phase was followed by a third one.

c) *Mass papers production*, the peak in Fig. 2. At the end of this phase, papers containing the energetic and SWaP considerations relevant to QI/QR finally appeared. The most relevant are: [76] "Current Readiness for Quantum Radar Implementation" and [56] "A system engineering perspective on quantum radar". These, and other, *"off chorus" publications* were very probably the origin of the present phase.

d) *Disillusionment and Endurance*. Since about 2020 the scientific literature [56], [57] and [70] clarified that implementing a QR for targets detection is an impossible task. A number of QR papers, including for example [51] and [55], does recognize that the laws of physics make it impossible to implement a Long Range QR (Long Range meaning: a Range above a few ten of metres). In practice, the Authors in the *"endurance"* period propose the use of QR/QI technology for new applications in the medical and biological area to continue their efforts in *quantum radars*. It is interesting to read the following in [55] (some boldface is added):

"We discuss the feasibility of using microwave quantum radars in biomedical sensing applications, with a heavy focus on the equipment needed to operate such radars. A recent microwave quantum radar experiment used a device called a Josephson Parametric Amplifier (JPA) to generate quantum entangled signals. Since ***JPAs or JPA-like devices are expected to be used in most quantum radar implementations in the foreseeable future****, we examine the operating requirements of JPAs. We find that the most important requirement for operating a JPA is the need for* ***cryogenic cooling****.* ***This is easily accommodated in medical environments such as hospitals****, so it is natural to consider whether quantum radars could work for biomedical sensing. We then find that, in fact,* ***many of the strengths of quantum radars are naturally suited to medical contexts.*** *We therefore conclude that the case for using* ***JPA-based quantum radars for medical sensing is a strong one****"*.
And:
"In fact, the size and power consumption of a JPA-based quantum radar are dominated by the requirements of the dilution refrigerator. The interior of such a refrigerator is shown in Fig. 2. When the helium tanks and other equipment required by a dilution refrigerator are considered, ***the total space needed is about the size of a large car. The power consumption is rather large, too: approximately 15 kilowatts****"*.
No further comments are needed, continuing the approach of *"a solution calling for a problem"*. This endurance approach reminds the proverb, in Padua language: *"Xe pèso el tacòn del buso"* i.e. *"the patch is worse than the hole"* for the reasons explained beforehand.
During the *"endurance"* period, some researchers seem not to accept the decreased interest of the scientific and operational community on Quantum Radar and wish to continue on this road. Some *"enduring"* researchers are trying to cancel the manifestation of opposite opinions whenever they can, for instance, when designated as anonymous Referees, as shown in Section 7.5.
Two final remarks are needed.

- All the afore-mentioned evaluations in this work refer to the most common radar applications aimed to detection and location of *"point"* targets such as aircraft. Of course, other radar applications, such as imaging, have their own specific power budget whose quantitative analysis, in the QR case, it outside the aim of this work. Among these particular applications, Microwave Tomography shall be noticed [104], [105], and [106]. As another example, in the Conclusions of [23] the QI (or QR) approach is prospectively mentioned to be useful for studies of cell biology, whenever the radiated power on single cell shall be constrained. However, the requested broadband operation hardly copes with the microwave (or radio-frequency) QI/QR technology because of the needed "phase matching" conditions [57] and the need for a few dB's of "Quantum Advantage" in those applications is very questionable.
- An increase of the theoretical Quantum Advantage of 6 dB's has been proposed in [107] where Hyperentangled photons are defined as simultaneously entangled in multiple degrees of freedom. In [107], it is proposed to use pairs of photons hyperentangled in two degrees of freedom, namely polarization and frequency-time, to achieve a $12\ dB$ performance improvement in the error probability exponent over the best-known quantum illumination procedure; the performance is theoretically evaluated for the particular case of $N_s = 0.01$, $N_B = 25$ and two-way attenuation $\kappa = 0.01$. The proposal refers to optical (Lidar) technology, not to microwave radar, and, even in the (undemonstrated) case of Hyperentangled QR really attaining the maximum Quantum Gain of $12\ dB$, the maximum radar Range values between 10 m and 40 m shall change into radar Range results between 20 m and 80 m which is substantially equivalent from an operational viewpoint.

Summing up, the title of [77]: *"The short, strange life of quantum radar"*, tells us, once again, that while *it is generally difficult to find the correct solution for a given problem*, often *it is even harder to find the correct problem for a given solution*.

7.5. Updates of Anti-Stealth and Quantum Radar

Still in the early 2025, after the well-documented critical papers that appeared in 2020 and followed in 2021-2024 (Section 7.3 and [109-114]) one may found papers on an alleged "anti-stealth" capability of a quantum radar, as well as LinkedIn posts like the following, dated 05/02/2025 and written by Kıvanç İnan (Ph.D.), senior principal engineer and Systems Engineering Director at the Company Aselsan (Ankara, Turkey):

*"Quantum radar is an emerging technology that uses the principles of quantum mechanics to improve the detection capabilities of radar systems. Unlike traditional radar, which relies on classical electromagnetic waves, quantum radar utilizes quantum entanglement and superposition to make objects detectable even in environments where traditional radar might fail, such as in the presence of strong interference or stealth technologies. It has the potential to dramatically improve range, accuracy, and the ability to detect low-profile targets, such as stealth aircraft or small drones. Quantum radar could be a game-changer for military, security, and aviation industries by overcoming the limitations of conventional radar systems. *Key Benefits:* - *Improved Detection of Stealth Objects:* Quantum radar could theoretically "see" through stealth technology, making it much harder for advanced military aircraft and drones to evade detection. - *Enhanced Sensitivity:* Quantum radar can detect faint signals, enabling it to work more effectively in challenging environments with a lot of background noise or interference. - *Future Applications:* Beyond military use, quantum radar may have applications in aviation safety, environmental monitoring, and autonomous vehicles*".

This text, written in good English, is fully false, as recognized using the common sense and by a few computations easily done by each radar engineer on the back of an envelope). Another example with similar content, dated 22 Jan. 2025, is found (on 07/02/2025) in https://cuashub.com/en/content/what-is-quantum-radar-and-how-will-it-change-drone-detection/#:~:text=Quantum%20radar%20is%20an%20emerging,even%20the%20most%20elusive%20targets.

In the, generally wise and careful, Wikipedia (https://en.wikipedia.org/wiki/Quantum_radar,

accessed on 07/02/2025) we find : "*There is media speculation that a quantum radar could operate at long ranges detecting stealth aircraft, filter out deliberate jamming attempts, and operate in areas of high background noise, e.g., due to ground clutter. Related to the above, there is considerable **media speculation** of the use of quantum radar as a potential anti-stealth technology*."

7.6. An industrial point of view

This study report is concluded by a contribution written by Frederick L. Daum from Raytheon, Principal Fellow at Raytheon, IEEE Fellow and Distinguished Lecturer for the IEEE.

Applications of Quantum Radar by F.L. Daum (please refer to [115], the IEEE Distinguished Lecture "Quantum Radar: Good Idea or Snake Oil?" dated 5 October 2021, freely available at https://vimeo.com/647062681 whose abstract follows: *We survey the latest theoretical results that quantify the benefits of quantum radar compared with optimal classical radars. We survey all of the known real world experiments for quantum radar. We analyze the cost of quantum radar compared with classical radars. We explore the question of what quantum radar is good for in the real world. We give an annotated list of a few good papers. We discuss the recent Chinese quantum radar demonstrations. We explain some of the practical limitations of quantum radar. We speculate about future research in quantum radar*).

There are no practical applications of quantum radar today, and there is no hope for any practical applications in the future. The reasons for this, listed from the most industrially relevant, are manifold: (1) QR costs many orders of magnitude more than comparable classical radars; (2)

the prime power for cooling QR would require many nuclear power plants; (3) there are many severe theoretical restrictions on QR that are not satisfied in practice; (4) the computational complexity for the optimal QR receiver would require a (non-existent) super-performant computer; (5) there are no practical advantages to use QR rather than classical radar; and (6) the best laboratory experiments with QR have yielded only order of 1 dB quantum advantage so far.

Nevertheless, a few diehards have suggested potential applications for QR, including medical imaging. Quantum radar researchers suggest medical imaging because it is an extremely short-range application (order of meter), and hence it would require less transmit power to obtain a useful detection and measurement. Also, such QR re-searchers assert that classical microwave radar power levels endanger the health of the patient for medical imaging. However, all of this is pure nonsense. The lack of any quantitative analysis in the QR papers advocating its use for medical imaging (or any other application) is telling. The US safety standard for microwave radiation for humans is 5 milliwatt per square cm, and similar, sometimes more conservative limits, apply in the rest of the word. We can make perfectly good medical images with a CR well below the allowed levels of power density. Cell phones radiate roughly one Watt of microwave power.

Some papers assert that QR would mitigate jamming and clutter and stealth, but all of these claims are also nonsense. There is a recent theoretical paper by that claims that QR can achieve range measurement accuracy that is better than the "old" classical radar. However, this paper not only assumes a never demonstrated 6 dB quantum advantage from using entanglement, but, most important, compares an optimal QR with a CR that is not allowed to use phase derived range (PDR). In particular, the paper assumes that the range measurement accuracy of a CR is based on the location of the peak amplitude of the envelope of the detected signal, and hence it is limited by the chirp bandwidth of the waveform. However, it is well known that a CR can achieve much better accuracy in range by using PDR.

Chinese engineers claimed that they had built a quantum radar that can track stealthy aircraft at 100 km range. However, this violates the laws of physics. A common error by Chinese engineers and by the physicists at Waterloo and Vienna is ignoring that if you amplify the transmitted signal by more than 3 dB, then you will destroy the quantum advantage.

It is easy to show that the prime power and cost required for a microwave QR would be astronomical. In particular, one cryogenic refrigerator requires about 10 kW of power for the cooling, but it produces only one attowatt of X-Band output power for a QR. This would be the most inefficient radar imaginable. For example, if you wanted to transmit 1 kW of X-Band power with your QR, then you would need about one trillion megawatts of prime power for the cooling of the cryogenic refrigerators. This follows from the simple back-of-the-envelope calculation. A typical nuclear power plant generates about one gigawatt of power. Hence, we would need about one billion nuclear power plants to provide the prime power for our 1 kW QR at X-Band, un-der the most optimistic assumptions (i.e., 100% bandwidth JPA). A radar with 1 kW of radiated power might be able to track large aircraft at short range (e.g., 10 km), assuming a reasonable size antenna. The Chinese claimed to track stealthy aircraft at 100 km range, and our calculation of prime power makes it obvious that the Chinese claim is bogus. If we wanted a very short range QR for medical imaging we might need a radiated power on the order of 1 mW. This would require about one thousand nuclear power plants to supply the prime power for cryogenic cooling; hence even very short range applications like medical imaging are completely out of the question.

The radiated power of a QR is not limited by technology, but rather it is limited by theory. In particular, a given QR device (e.g. JPA) cannot transmit more than one aver-age photon per optical mode in order to have any quantum advantage over a classical radar. Furthermore, we cannot amplify the transmitted signal of a QR by more than 3 dB and obtain any quantum advantage. Both of these are theoretical limitations, as explained in [52].

In summary, there is no hope for any practical applications of quantum radar, owing to overwhelming issues of cost, prime power requirements, impractical theoretical limitations, huge computational complexity and lack of any practical advantage over classical radars. These fundamental problems cannot be overcome by improved engineering or more research, but rather one would have to increase the numerical val-ue of Planck's constant by many orders of magnitude.

We are not the only engineers who hold such opinions about QR. In particular, the Defense Science Board (DSB) of the US Pentagon publicly announced a few years ago that QR "will not provide upgraded capability to DOD (Department Of Defence)". This is the same DSB report that strongly encouraged continued research and development of quantum communication, quantum navigation, quantum clocks, quantum metrolo-gy, and even quantum computing. Moreover, serious QR researchers have emphatically pronounced QR to be a bad idea for practical applications. Unfortunately, such honest scientists are drowned out by the chorus of bogus videos and new articles and erroneous technical papers and click bait which abound on the internet. The amount of hype and misinformation concerning QR is truly surprising; it even exceeds the hype about quantum computing, which is driven by companies trying to make money from it, as well as venture capitalists attempting to recoup their investments. Ego, technical incompetence, and fraud also play substantial roles in the proliferation of QR hype.

Perhaps we should not be too surprised, because the number of people who understand both quantum mechanics and radar systems is rather small. There are many smart physicists and radar system engineers who still labour under the illusion that QR is worth further research and development. They erroneously think that the problems with QR are technological rather than theoretical. They suppose that more money would solve such technological problems, analogous to the spectacular development of powered flight in the 1900's starting with the Wright brothers' nascent work.

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